OPTIMAL DESIGN OF DISC CAM MECHANISMS

By

TUKARAM GANAPATI KALE

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

APRIL, 1981

OPTIMAL DESIGN OF DISC CAM MECHANISMS

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

By

TUKARAM GANAPATI KALE

to the
DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

APRIL, 1981

Dedicated

to my teacher

(Late) Shri Madhukar (Appa) Karmarkar

22.4.81

CERTIFICATE

This is to certify that the work entitled 'Optimal Design of Disc Cam Mechanisms' by Tukaram Ganapati Kale has been carried out under my supervision and has not been submitted elsewhere for the award of a degree.

(Dr. S.G. Dhande)
Assistant Professor
Department of Mechanical Engineering
Indian Institute of Technology
Kanpur-208016.

April, 1981

ACKNOWLEDGEMENTS

I am indebted to my teacher Dr. S.G. Dhande for introducing me to the area of Computer Aided Design and Computer Graphics. I express my deep sense of gratitude to him for his excellent guidance and encouragement throughout the period of the thesis work.

I am thankful to M/s. C. Rameshchandran, N. Rajaram, P.P. Thankachan (Tank), J.B. Bhattacharjee, P.G. Poonacha, S. Datta for their valuable help during my stay in the campus.

Finally, I wish to thank to Mr. C.M. Abraham for the skillful typing.

TUKARAM G. KALE

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

a	- offset distance between the pivots of cam and follower
$a_1, a_2, \ldots, a_n$	- design variables
E_1, E_2	- moduli of elasticity of contacting surfaces
l	- arm length of follower
$\underline{n}^{(1)}$	- normal to surface Σ_1 at point of contact
$\underline{n}^{(P)}$	- normal vector to surface at point P
P	- point of contact
p_1	- follower displacement (linear or oscillating)
$p_1^l,\ p_1^u$	- limiting values of p_1
p_2	- angular rotation of cam
r	- radius of roller
r_b	- base radius
$S(X,O,Y)$ $S_1(X_1,O_1,Y_1)$ $S_{\bar{1}}(X_{\bar{1}},O_{\bar{1}},Y_{\bar{1}})$ $S_2(X_2,O_2,Y_2)$ $S_{\bar{2}}(X_{\bar{2}},O_{\bar{2}},Y_{\bar{2}})$	- co-ordinate systems
s_o	- initial position of follower
s_1	- linear displacement of follower
s_1'	- derivative of s_1 with respect to ϕ_2
s_1''	- derivative of s_1' with respect to ϕ_2
$\underline{v}^{(1)}$	- velocity vector
w_2	- angular velocity of cam

β	-	angle representing the position of the follower arm with respect to reference line
β_{lower}	-	lower limit on β
β_{upper}	-	upper limit on β
η	-	r/l
η_{lower}	-	lower limit on η
η_{upper}	-	upper limit on η
ζ	-	a/l
λ	-	r_b/l
θ	-	angle representing the position of point of contact with respect to reference line
θ_{lower}	-	lower limit on θ
θ_{upper}	-	upper limit on θ
θ_{min}	-	minimum value of θ
θ_{max}	-	maximum value of θ
ϕ_1	-	angular rotation of follower
ϕ_2	-	angular rotation of cam
ρ	-	radius of curvature
ρ_1, ρ_2	-	radii of curvature of contacting surfaces
ρ_{min}	-	minimum value of radius of curvature
ρ^*	-	limiting value of radius of curvature
ψ	-	pressure angle
ψ_{max}	-	maximum value of pressure angle
ψ^*	-	limiting value of pressure angle
χ	-	curvature of the cam profile
μ_1, μ_2	-	poisson's ratios for contacting surfaces
Σ_1, Σ_2	-	notations for contacting surfaces

$\|\ldots\|$	- absolute value of the enclosed expression
$\|\|\ldots\|\|$	- Euclidean norm of the enclosed vector
$\|\ldots\|^T$	- transpose of the enclosed vector
$\underline{R}^{(P)}\|X,Y,1\|^T$	- radius vector of point P in fixed co-ordinate system
$\underline{R}_1^{(P)}\|X_1,Y,1\|^T$	-
$\underline{R}_2^{(P)}\|X_2,Y_2,1\|^T$	- radius vector of point P in moving co-ordinate systems of the follower and the cam respectively
$C\theta \equiv \cos\theta$	-
$S\theta \equiv \sin\theta$	

ABSTRACT

In the present work a procedure has been developed for optimal design of disc cam mechanisms. The present approach is based on only kinematic constraints. Two main kinematic constraints taken into account are (i) the maximum value of the pressure angle and (ii) the minimum value of the radius of curvature. Analytical expressions for the pressure angle and the radius of curvature have been used. Numerical schemes have been developed to evaluate these constraints. The design procedure finds different sets of the design variables which satisfy the above-mentioned two constraints. Then that set of design variables is chosen which gives minimum size cam. Following types of cam follower mechanisms are considered; (i) a disc cam with a translating, roller-ended follower, (ii) a disc cam with an oscillating, roller-ended follower. Finally, two numerical examples of each type have been solved.

Chapter 1

INTRODUCTION

1.1 Introduction

Mechanisms such as cams and linkages are used for moving machine-components according to prescribed motion specifications. In case of linkages, the prescribed motion specifications are achieved by proportioning the lengths of the constituting links. In case of cam mechanisms the prescribed motion specifications are achieved by selecting appropriate shapes of contacting surfaces attached to the links transmitting motion from the input link to the output link.

Because of the flexibility of selecting a pair of conjugate enveloping curves (or a pair of surfaces) as the cam and the follower profiles, it is always possible to produce a variety of motion specifications for the output link of a cam mechanism. This is a distinct advantage in choosing a cam mechanism over linkages. Therefore, cams have found wide applications in almost all types of machinary, for example, textile machines, packaging machines, machine tools, computer peripherals etc. .

Design of cam mechanism is generally based on kinematic input-output specifications. In general, the follower profile is assumed to be in the form of a circular roller or a flat

plane (for disc cams). Using the principles of envelope theor[1,2], it is possible to find a parametric equation of the cam profile, when the follower profile-geometry is given along wit the specifications of the input-output motions.

When cam and follower profiles are designed for transmitt a prescribed motion, basic objective of cam design is fulfille However, it is necessary to ensure that the performance of suc a cam mechanism is also satisfactory from other operational co straints. Two salient constraints controlling the performance a cam mechanism are as follows :

(i) The kinematic efficiency of the higher pair formed by the contacting profiles should be as high as possible. and (ii) th contact stress at the point of contact should be as low as possible.

The magnitudes of the kinematic efficiency and the contac stress , in turn, depend on the geometry of the cam profile. For instance, the kinematic efficiency can be measured by the magnitude of the pressure angle and the contact stress primari depends on the radius of curvature of the cam profile at the point of contact. It is, therefore, essential not only to sel a parametric equation of a cam profile but also to select othe parameters such as the base radius, the offset distance, the roller radius, the centre distance between the cam and the follower pivots etc. in such a way that critical values of the controlling parameters, namely, the pressure angle and the rad of curvature of the cam profile are within specified limits.

In practice it is always desired to have a cam mechanism

that the maximum pressure angle is less than or equal to a prescribed maximum value and the minimum radius of curvature is above a certain prescribed minimum value. Furthermore, in some cases, it is necessary to design a cam mechanism which not only provides the prescribed input-output considerations but also provides an optimal design (or configuration) which gives an extremum value of an objective function. Many a times, the size or the weight of the components of a cam mechanism are minimised.

The objective of the present work is to propose a scheme such that the enveloping profiles do produce the prescribed input-output motion and other parameters such as the base radius, the offset distance etc. are selected in such a way that the operational constraints on the pressure angle and the radius of curvature are satisfied in the best possible way. The proposed scheme produces a number of combinations of design parameters. Finally, the optimal solution based on the size or the weight constraint can be obtained by search and selection method.

The present approach is based on only kinematic considerations. Dynamic considerations have not been taken into account.

1.2 Literature Survey

Since the kinematics of a cam mechanism depends on the

geometry of the contacting surfaces, most of the literature available for determination of the pressure angle and the radius of curvature is by use of graphical constructions. Notable among them are Rothbart [3], Jensen [4], Kloomak and Muffley [5,6,7], Gaunter and Uicker [8] C. Mischke[9] and J. Chakraborty and S.G. Dhande [10]. Rothbart [3] describes a graphical construction for finding the maximum pressure angle. The assumption is made that the maximum pressure angle occurs at the point of the maximum velocity. With this assumption the maximum pressure angle depends on two factors - the cam size and the basic input-output motion curve. The graph of the maximum pressure angle versus cam rotation is plotted for each basic curve. For most of the disc cam design the above mentioned assumption is the best approximation but in general, this gives error for small cams. Jensen [4] has prepared nomograms to find the maximum pressure angle and the minimum radius of curvature. There are different sets of nomograms for different motion curves. The nomograms are prepared for the radial translating followers but they can be used with good approximation for the offset and swinging roller-ended followers. The nomograms are prepared on the assumption that the maximum pressure angle occurs at the point where the velocity is maximum. One important reference is a series of papers by Kloomak and Muffley [5,6,7]. A series of design charts are presented. To use these charts designer first has to select the design

parameters such as, the base radius, the roller radius etc. and then using these charts one can find the maximum pressure angle and the minimum radius of curvature of the cam profile. If the pressure angle and the radius of curvature thus found are not within the prescribed limits, then change the values of one or all the design parameters and thus one can iterate on a variety of design decisions without plotting a graphical layout for each iteration.

Construction of these charts provided a tremendous achievement from the point of view of extensive calculations involved. Each point on the curves of these design charts is a result of extensive search done on the digital computer for a particular set of design parameters.

Further Gaunter and Uicker [8] repeated similar calculations and search procedures described by Kloomak and Muffley with the use of more efficient modern digital computers. This work removed a number of errors in the earlier design charts.

All the above works give the procedures for design of a disc cam mechanism. But one has to do a number of iterations before arriving at a particular set of design parameters such that, the operational constraints namely the pressure angle and the radius of curvature are satisfied in best possible way. C. Mischke[9] has described a graphical construction to find an optimal offset distance on a translating follower plate cam. A

suboptimization procedure is presented which determines the minimum pitch circle radius without use of the digital computer.

J. Chakraborty and S.G. Dhande [10] have developed a better approach to the analysis of geometric and kinematic parameters of the planar and spatial cam mechanisms. The expressions developed for the pressure angle and the radius of curvature are of parametric type. Using some numerical techniques these expressions can be solved and thus search for the maximum pressure angle and the minimum radius of curvature can be done on digital computer.

1.3 Pressure Angle Considerations

Pressure angle is an index of the kinematic efficiency of an input-output device, in general, and of a cam-and-follower mechanism, in particular. In order to define the pressure angle in a most general way, consider a cam and follower mechanism as shown in Fig. 1.1. At any given instant consider that cam link 2 is transmitting motion to the follower link 1 through a pair of conjugate surfaces Σ_2 and Σ_1 attached to the links 2 and 1 respectively. Moreover P is the point of contact between Σ_2 and Σ_1. A common normal $\underline{n}^{(1)}$ indicates the direction of the reactive force which will try to move the follower link according to kinematic constraint of the mechanism. If $\underline{v}^{(1)}$ represents the velocity vector along which point P when considered attached to link 1 is moving at that instant, it is clear that

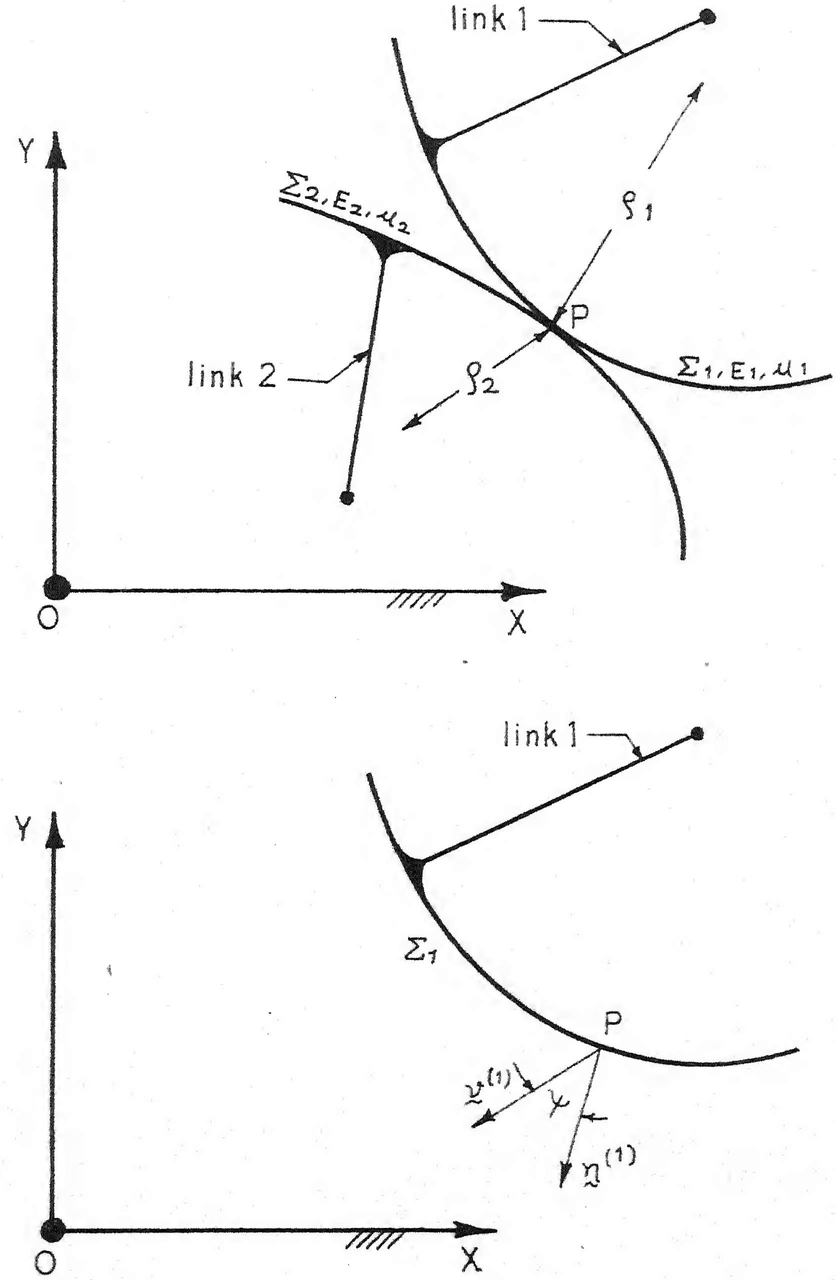

link 1
Y
Σ_2, E_2, μ_2
ρ_1
P
link 2
ρ_2
Σ_1, E_1, μ_1
O
X
link 1
Y
Σ_1
P
$\nu^{(1)}$
χ
$n^{(1)}$
O
X

FIG. 1.1

not all the reactive force acting along $\underline{n}^{(1)}$ will be able to move link efficiently. As a matter of fact since the directions of $\underline{n}^{(1)}$ and $\underline{v}^{(1)}$ at point P are different only a part of reactive force will be able to move the link along $\underline{v}^{(1)}$. The fraction representing the effective component of the reactive force responsible for producing $\underline{v}^{(1)}$ will depend on the acute angle between the directional vectors $\underline{n}^{(1)}$ and $\underline{v}^{(1)}$. An analytical expression for pressure angle ψ is given by [10],

$$\cos \psi = \frac{|\underline{n}^{(1)} \cdot \underline{v}^{(1)}|}{||\underline{v}^{(1)}||} \qquad (1.1)$$

where $\underline{n}^{(1)}$ and $\underline{v}^{(1)}$ are expressed in the global refrance co-ordinate system XOY.

The magnitude of the pressure angle is important for the following reasons.

(i) Larger the value of the pressure angle, it gives more side thrust and this increases the forces created on the cam-follower mechanism.

(ii) Lower the value of the pressure angle, it gives larger size of the cam and is not desirable for following reasons.

   (a) The size of the cam gives to a certain extent, the size of the machine.

   (b) Larger size cam gives higher circumferential speed. Then small errors in the profile of the cam will cause additional acceleration.

(c) Larger size cams give more revolving weight, which increases vibrations in the machine if it happens to be eccentric.

1.4 Radius of Curvature Considerations

At any given instant, the radius of curvature of the contacting surfaces at a point of contact is a measure of compressive stress developed at that point. Two cylindrical surfaces in contact and in allignment produce a maximum compressive stress below the surface and its magnitude is given by [3]

$$S_c = C \left[\frac{F\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)}{t_h\left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right)} \right]^{1/2} \qquad (1.2)$$

where

S_c - maximum compressive stress at any point,

C - constant,

F - normal load,

ρ_1 and ρ_2 - radii of curvature of the contacting surfaces,

t_h - thickness of the contacting surfaces,

μ_1 and μ_2 - Poisson's ratio for the contacting surfaces,

E_1 and E_2 - moduli of elasticity of the contacting surfaces.

For a given configuration the radius of follower is constant. The value of the thickness t_h depends on the rigidity of the cam shaft.

At any speed, the load and cam radius of curvature are changing along the profile. Thus the stress is different at every point on the cam. The point on the cam profile where the radius of curvature is minimum represents the largest compressive stress.

The minimum radius of curvature of the cam profile should be kept as high as possible for the following reasons.

i) To prevent the under cutting of the convex portion of the cam,

ii) To prevent high contact stresses.

1.5 Scope of Present Work

The objective of the present work is to develop the procedure for the design of disc cams taking into account the constraints on the maximum value of the pressure angle and the minimum value of the radius of curvature. Analytical expressions for the pressure angle and the radius of curvature have been used. These expressions are transcendental, parametric equations; the parameter being the input angular rotation of the cam. Since it is extremely difficult to derive exact analytical expression for the angular rotation of the cam at which either the pressure angle is maximum or the radius of curvature is minimum, numerical schemes have been proposed in the present work.

The design procedure starts with taking into account only the pressure angle constraint. Number of alternative design solutions are found for the condition that the maximum pressure angle during one complete cycle of cam rotation is equal to the allowable value. Out of these alternative designs, those which do not satisfy the radius of curvature constraint are eliminated. The remaining set forms a group of feasible solutions. If no any feasible solution is found which satisfies the pressure angle constraint critically and also satisfies the radius of curvature constraint, then a design sub-critical from the pressure angle constraint viewpoint but feasible from the radius of curvature constraint viewpoint is found. Minor variations in this procedure have been made depending on whether the follower motion is translating or oscillating. Detailed discussion of the scheme is presented in Chapter 2. Numerical examples illustrating the scheme of design are given in Chapter 3 and Chapter 4 for the translating follower case and the oscillating follower case respectively. Conclusions drawn from numerical experimentation done during the course of this work are given in Chapter 5.

Chapter 2

OPTIMAL DESIGN FORMULATION

2.1 Statement of the Problem

The problem of cam design in the present investigation can be stated as follows.

It is required to design a two dimensional disc cam driving a translating or an oscillating roller - ended follower having the size of the cam as minimum as possible and satisfying the following constraints.

$$\psi_{max} \leq \psi^* \tag{2.1a}$$

and

$$\rho_{min} \geq \rho^* \tag{2.1b}$$

where,

ψ_{max} is the maximum value of the pressure angle during one complete rotation of cam,

ψ^* is the limiting value of the pressure angle (preassigned),

ρ_{min} is the minimum value of the radius of curvature during one complete rotation of cam and

ρ^* is the limiting value of the radius of curvature (preassigned).

2.2 Analytical Solution Procedure

The expressions for the pressure angle, ψ and the radius of curvature, ρ can be represented as,

$$\psi = \psi(a_i,\ i = 1,2,3,\ \ldots\ ,\ n,\ p_1\ (p_2),\ \frac{dp_1}{dp_2},\ p_2) \tag{2.2a}$$

and

$$\rho = \rho(a_i,\ i = 1,2,3,\ \ldots\ ,\ n,\ p_1\ (p_2),\ \frac{dp_1}{dp_2},\ \frac{d^2p_1}{dp_2^2},\ p_2) \tag{2.2b}$$

where, $a_1, a_2, a_3, \ldots, a_n$ are design variables,

p_1 is the follower displacement (linear or angular) with limiting values p_1^l and p_1^u,

$$p_1^l \leq p_1\ (p_2) \leq p_1^u\ ,$$

p_2 is the angular rotation of cam,

and

$$0 \leq p_2 \leq 2\pi\ .$$

The range of variable p_2 (i.e. one complete rotation of cam), is divided into different sections such as Rise, Dwell, Fall, etc.. If $p_{2,j}^l$ and $p_{2,j}^u$ are the limiting values of p_2 in the section j, then

$$p_{2,j}^l \leq p_2 \leq p_{2,j}^u\ , \quad \text{for } j = 1,2,\ \ldots,\ k$$

and

$$p_{2,1}^l = 0.,\qquad p_{2,k}^u = 2\pi,$$

$$p_{2,j}^u = p_{2,(j+1)}^l,\ \text{for } j = 1,2,\ \ldots,\ (k-1).$$

According to the requirement of the problem, input-output motion curves are selected and the range of p_2 is divided into different sections. Then p_1, $\frac{dp_1}{dp_2}$, $\frac{d^2p_1}{dp_2^2}$ can be written in terms of p_2, resulting in

$$\psi = \psi(a_{i,i=1,2,3,\ \ldots,\ n,p_2}) \quad \text{and} \tag{2.2c}$$

$$\rho = \rho(a_{i,i=1,2,3,\ \ldots,\ n,p_2}) \tag{2.2d}$$

To find the maximum value of the pressure angle, $_{max}$

$$\frac{\partial\psi}{\partial a_i} = 0; \quad i = 1,2,3,\ \ldots,\ n,$$

$$\frac{\partial\psi}{\partial p_2} = 0$$

Solving these (n+1) equations simultaneously a solution,

$(a_{i*}\ ,\ i = 1,2,3,\ \ldots,\ n,\ p_{2*})$, can be obtained.

In general, we can call this set as X^*.

Then for value of the pressure angle at X^* to be a maximum, the Hassian matrix $J|_{X = X^*} = [\frac{\partial^2\psi}{\partial a_i\ \partial a_j}|_{X = X^*}]$ should be negative definite. Similarly, following conditions need to be satisfied so as to find the minimum value of the radius of curvature.

$$\frac{\partial\rho}{\partial a_i} = 0; \qquad i = 1,2,3,\ \ldots,\ n\ ,$$

$$\frac{\partial\rho}{\partial p_2} = 0.$$

Solving these (n+1) equations we get different sets of variables,

$$(a_{i**},\ i = 1,2,3,\ \ldots,\ n,\ p_{2**}) \text{ or in general, } X^{**}$$

So for minimum value of the radius of curvature, ρ_{min} the Hassian matrix $J|_{X = X^{**}} = [\frac{\partial \rho}{\partial a_i \, \partial a_j} |_{X=X^{**}}]$ should be positive difinite.

In general, expressions for ψ and ρ are transcendental and highly non-linear. These expressions, when extended turn out to be algebraic polynomials of degree varying from eighth to sixteenth depending on the choice of the input-output curves.

Following are the difficulties in adopting the above mentioned analytical approach to solve the present problem.

(i) Since the expression for ψ and ρ are transcendental and non-linear it is difficult to derive analytical expressions for $\frac{\partial\psi}{\partial a_i}$, $\frac{\partial\rho}{\partial a_i}$, $\frac{\partial\psi}{\partial p_2}$ and $\frac{\partial\rho}{\partial p_2}$.

(ii) Analytical solution of the equations involving these derivatives are required to locate the extremum value. Since these derivatives are polynomials of high degree, it is difficult to pursue the analytical approach. Hence numerical solution procedures are selected.

2.3 Numerical Solution Procedures

The problem of finding extremum values of Pressure Angle and Radius of Curvature can be considered as a non-linear

optimization problem. However, the constraints involved are of parametric type. Unfortunately elegant schemes of solving non-linear optimization problems having parametric constraints are not available [11].

In the present work, the following procedure has been adopted.

Step 1 : Select a set of trial values for the design variables, namely, the offset distance, the centre distance of pivots etc..

Step 2 : Once the values of the design variables are known, the equation $\frac{d\psi}{dp_2} = 0$ is solved using any numerical scheme such as the Direct Search method or the Newton-Raphson method. The Hessian matrix is also tested for negative definiteness at every stage of numerical iteration.

Step 3 : Find the value of the pressure angle, ψ_{max}, at the value of p_2, found as the solution of the numerical scheme of Step 2.

Step 4 : Compare the computed value of ψ_{max} with ψ^*, the limiting value.

Step 5 : If $|\psi_{max} - \psi^*| > \varepsilon_1$, where ε_1 is as small number as possible, then reassign the values of one or more design variables and go to Step 2.

Step 6 : If $|\psi_{max} - \psi^*| \leq \varepsilon_1$, then solve the equation $\frac{d\rho}{dp_2} = 0$ and find the value of ρ_{min} using the similar technique to solve $\frac{d\rho}{dp_2} = 0$.

Step 7 : If $| \rho_{min} - \rho^* | \geq \varepsilon_2$, where ε_2 is as small number as possible, then declare this as a feasible design. If this condition is not satisfied then reassign the values of some or all the variables and go to Step 2.

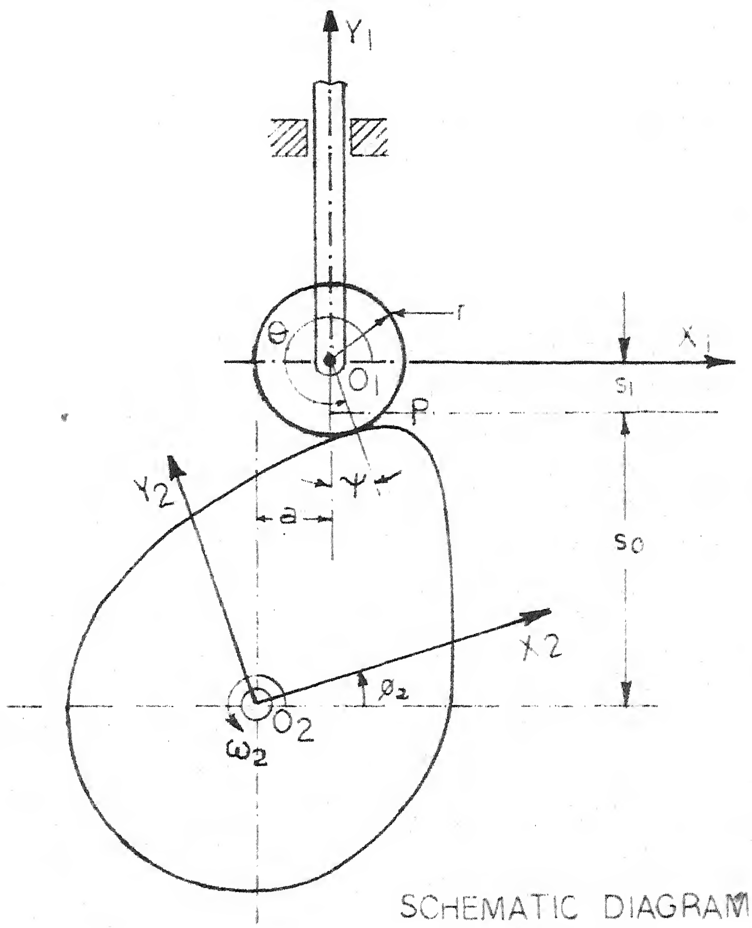
Step 8 : Find different sets of design variables. Check the size of the cam and select that particular set of design variables which gives the smallest size cam.

Chapter 3

DESIGN OF A DISC CAM WITH A TRANSLATING, ROLLER-ENDED FOLLOWER

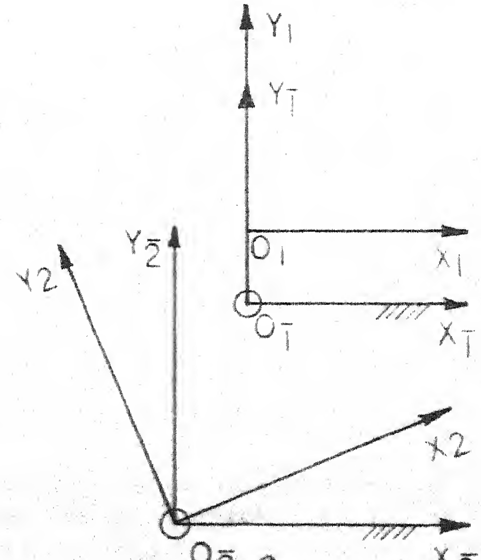
3.1 Design Procedure

In this section a procedure of design of a disc cam with a translating,roller-ended follower is discussed. The design procedure takes into account both the pressure angle constraint as well as the radius of curvature constraint. Fig. 3.1a shows a schematic diagram of the cam mechanism under discussion. Fig. 3.1b shows the orientation of various co-ordinate systems used in the analysis. The co-ordinate system $S_1(X_1, O_1, Y_1)$ is attached to the follower and the co-ordinate system $S_2(X_2, O_2, Y_2)$ is attached to the cam. Initial positions of the co-ordinate systems S_1 and S_2 are designated as $S_{\bar{1}}$ $(X_{\bar{1}}, O_{\bar{1}}, Y_{\bar{1}})$ and $S_{\bar{2}}(X_{\bar{2}}, O_{\bar{2}}, Y_{\bar{2}})$ respectively. A global co-ordinate system, fixed in the space, is designated by $S(X,O,Y)$ and, in the present case, $S_{\bar{1}} \equiv S$. The cam rotates with a uniform angular velocity w_2. The input parameter of the motion of the cam is the angular rotation of the cam, ϕ_2. The linear displacement of the follower from its initial position is the output parameter of the motion and is denoted by s_1. The initial position of the follower is given by the distance s_o. Distance 'a' denotes the offset distance between the pivot of the cam and the line of travel of the follower. At any instant, given by



SCHEMATIC DIAGRAM

FIG. 3·1a



CO-ORDINATE SYSTEM

FIG. 3·1b

the angular rotation of the cam ϕ_2, P denotes the point of contact between the follower surface and the cam surface. The displacement of the follower s_1 is a function of ϕ_2 and is given by

$$s_1 = s_1(\phi_2) \tag{3.1}$$

The unit normal vector $\underline{n}^{(P)}$ at the point P on the follower surface is given by [10],

$$\underline{n}^{(P)} = |C\theta,\ S\theta,\ 0\ |^{T*} \tag{3.2}$$

where θ is measured from X axis and denotes the position of point of contact at the given instant. The parametric equations for the follower roller surface, the cam profile, the pressure angle, the radius of curvature and the condition of contact are derived in [10] and as follows :

(i) The follower roller surface

$$\underline{R}^{(P)} = |r\ C\theta,\ r\ S\theta,\ 1\ |^{T} \tag{3.3}$$

where r is the radius of the roller.

(ii) The cam profile

$$\underline{R}_2^{(P)} = |x_2,\ y_2,\ 1|^{T} \tag{3.4}$$

$$x_2 = r\ C(\theta - \phi_2) + aC\phi_2 + (s_o + s_1)\ S\phi_2$$

$$y_2 = r\ S(\theta - \phi_2) - aS\phi_2 + (s_o + s_1)\ C\phi_2$$

*When no subscript is attached to a vector it is presumed that, the vector is expressed in the global co-ordinate system S(X,O,Y).

(iii) The condition of contact, θ

$$\theta = \tan^{-1}\left(\frac{s_o + s_1}{a - s_1'}\right) \qquad (3.5)$$

(iv) The pressure angle,

$$\psi = \cos^{-1}\left(|S\theta|\right) \qquad (3.6)$$

(v) The radius of curvature, ρ

$$\rho = \frac{1}{\chi} \qquad (3.7)$$

where χ is curvature and

$$\chi = -\frac{1}{r}\,(1+(S\theta(s_o+s_1) + C\theta(a-s_1'))^2/(r(S\theta\, s_1'' + C\theta\, s_1') - (S\theta(s_o+s_1) + C\theta(a-s_1'))\,(r+S\theta(s_o+s_1) + C\theta(a-s_1'))))$$

General design procedure is discussed in the section 2.3 of Chapter 2. For the roller-ended translating follower case the design variables are,

(i) the roller radius, r,

(ii) the offset distance, a and

(iii) initial displacement of the follower, s_o.

The design procedure, given in the form of a flow chart in Fig. 3.2, is as follows :

Step 1 : Choose input-output motion curves according to the requirements of the problem. Read the values of a_{lower}, a_{upper}, ε_1, step a, step s_o, ψ^*, ρ^*.

Step 2 : Assign the value of $a = a_{lower}$.

Step 3 : Assign the value of a = a + step a and $s_o = 0$.

Step 4 : Compare the values of a and a_{upper}. If a is greater than a_{upper}, go to Step 13.

Step 5 : Assign the value of $s_o = s_o$ + step s_o.

Step 6 : Find the value ψ_{max}.

Step 7 : Compare the values of ψ_{max} and ψ^*. If ψ_{max} is greater than ψ^* go to Step 5.

Step 8 : Find the value of ρ_{min}.

Step 9 : Compare the values of ρ_{min} and ρ^*. If ρ_{min} is less than ρ^* go to Step 5.

Step 10 : Compare the values of step s_o and ε_1. If step s_o is less than or equal to ε_1 go to Step 12.

Step 11 : Assign the value of $s_o = s_o$ - step s_o and step s_o = step $s_o/2$. Go to Step 5.

Step 12 : Find the size of cam and go to Step 3.

Step 13 : Compare the different sizes of cam and choose the smallest size of cam.

Step 14 : Print the results of the design of this smallest size cam.

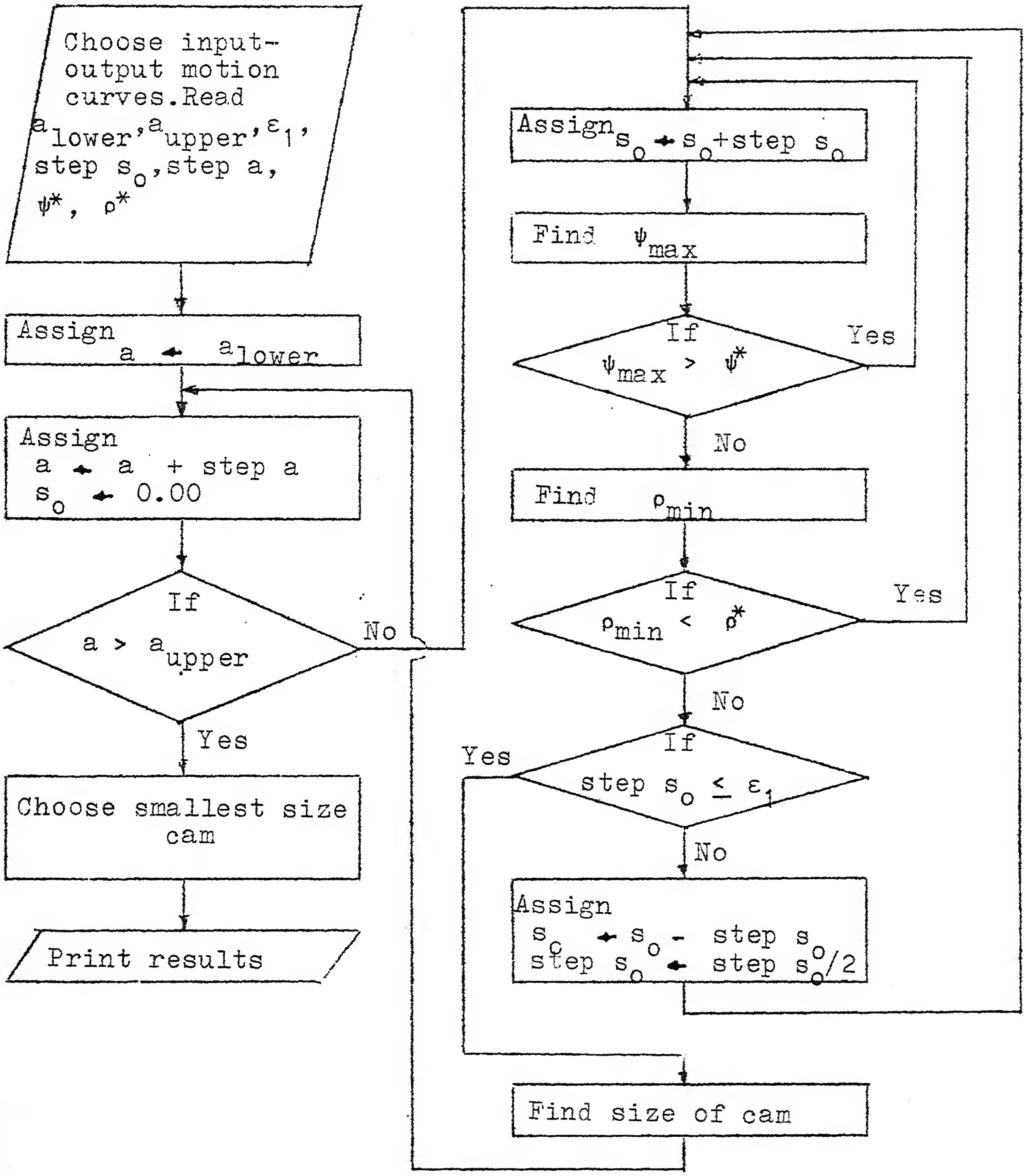


Fig. 3.2 Flow Chart

3.2 Numerical Examples

Example No.1

This example illustrates the design procedure applied to a disc cam with a translating roller-ended follower. The input data is given in Table 3.1. The output is presented in Table 3.2. The design procedure starts with an offset distance 'a' = 1.0 unit. Using the design scheme given in the Section 3.1 the magnitude of the initial displacement of the follower, s_o is found. The same design procedure is repeated with a value of 'a' changed from its previously assumed value by an increment of -0.1 units. The iterations are continued till the case when 'a' becomes equal to -1.0 unit. Each time magnitude of the base radius vector, r_b which is equal to $\sqrt{a^2 + s_o^2}$ is computed. Comparing the different values of r_bs for different cases of 'a', we select that combination of design variables a and s_o which gives the smallest value of r_b. It is presumed that the size of the cam is governed directly by the value of r_b. The final results are given in Table 3.3.

Fig. 3.3 shows the variation of the pressure angle, ψ . Fig. 3.4 shows the variation of the radius of curvature, ρ . Cam profile is shown in Fig. 3.5.

In this example the radius of roller is kept constant and equal to 1.0 unit. The limiting value of the maximum pressure angle, ψ^* is 30.0^o. The limiting value of the

minimum radius of curvature, ρ^* is 2.0 units. Fig. 3.3 shows that the pressure angle constraint is not critically satisfied. The radius of curvature constraint is satisfied critically which is seen clearly in Fig. 3.4.

Table 3.1 Input Data of Example No.1

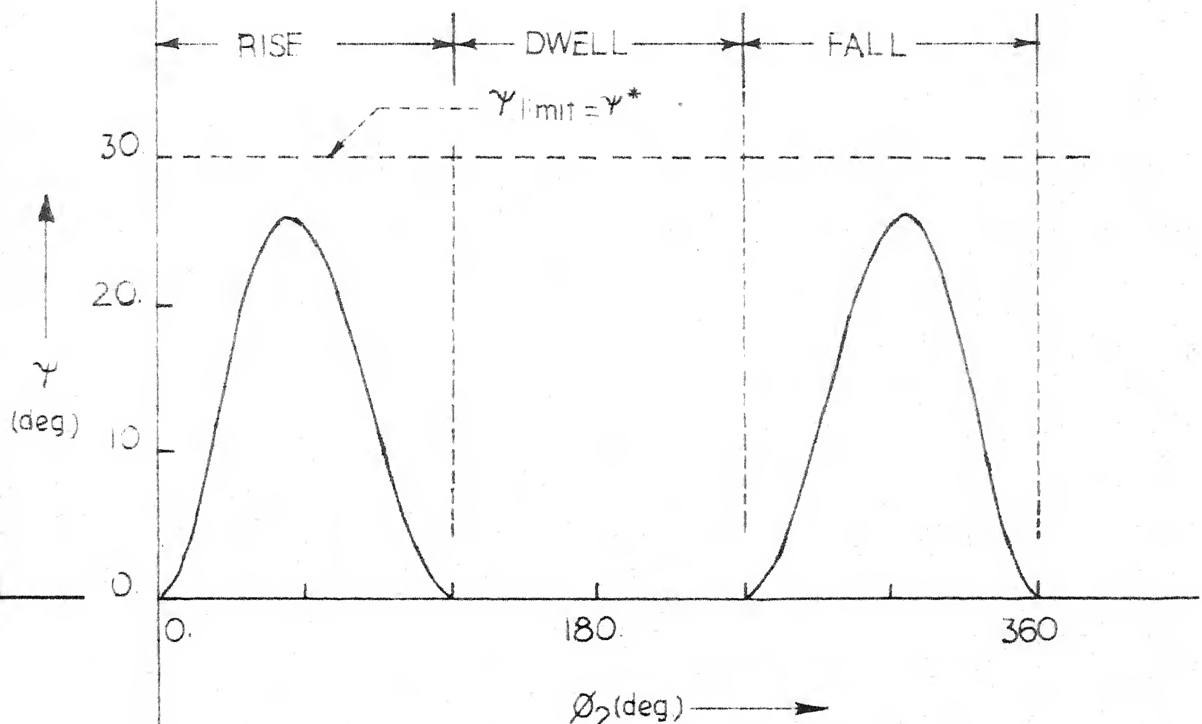
1.	Roller radius	r	1.0 unit
2.	Limitting value of the maximum pressure angle	ψ^*	30.0°
3.	Limiting value of the minimum radius of curvature	ρ^*	2.0 units
4.	Kinematic Data :		

Range of ϕ_2 (in degree)	Range of s_1 units	Total Displacement units	Input-Output Motion Curve
0 - 180	0.00 - 2.00	2.00	Full cycloid
180 - 240	2.00 - 2.00	0.00	-
240 - 360	2.00 - 0.00	-2.00	Full cycloid

Note : Detailed equations for input-output motion curves are given in Appendix I.

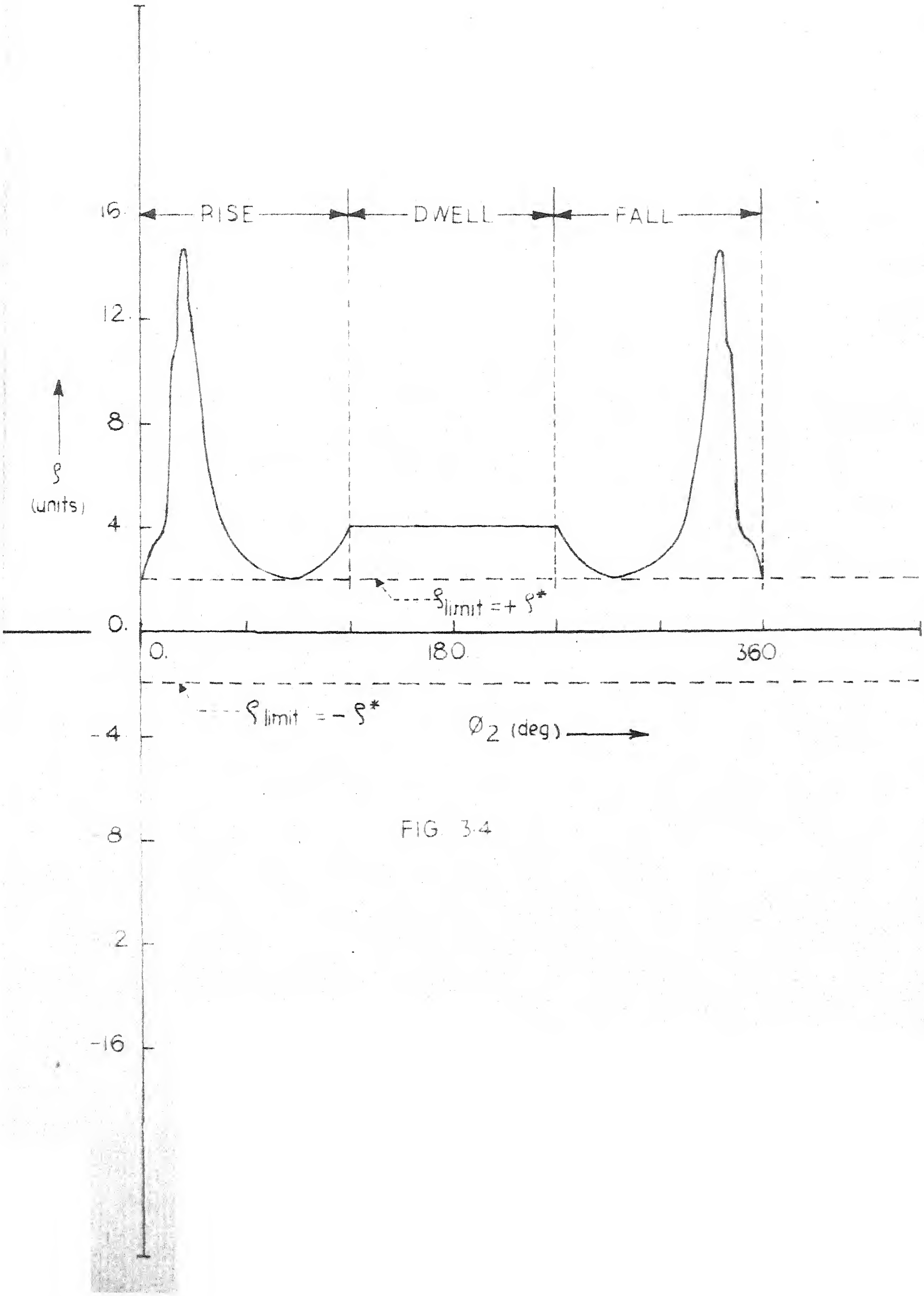
Table 3.2 Results of the Design Scheme for Example No. 1

Offset Distance, a units	Initial displacement of follower, s_o units	Base radius vector, r_b units
1.000	4.188	..4.305
0.900	4.000	4.100
0.800	3.844	3.926
0.700	3.656	3.723
0.600	3.469	3.520
0.500	3.313	3.350
0.400	3.125	3.151
0.300	3.031	3.042
0.200	3.031	3.038
0.100	3.000	3.002
-0.000	3.000	3.000
-0.100	3.000	3.002
-0.200	3.031	3.038
-0.300	3.031	3.042
-0.400	3.125	3.151
-0.500	3.313	3.350
-0.600	3.469	3.520
-0.700	3.656	3.723
-0.800	3.844	3.926
-0.900	4.000	4.100
-1.000	4.188	4.305



RISE
DWELL
FALL
$\psi_{limit} = \psi^*$
30
20
10
0
ψ
(deg.)
0.
180
360
ϕ_2(deg.)

FIG.3.3



RISE
DWELL
FALL
16
12
8
4
0.
-4
8
2
-16
ρ (units)
$\rho_{limit} = +\rho^*$
$\rho_{limit} = -\rho^*$
0.
180
360
ϕ_2 (deg)

FIG. 3.4

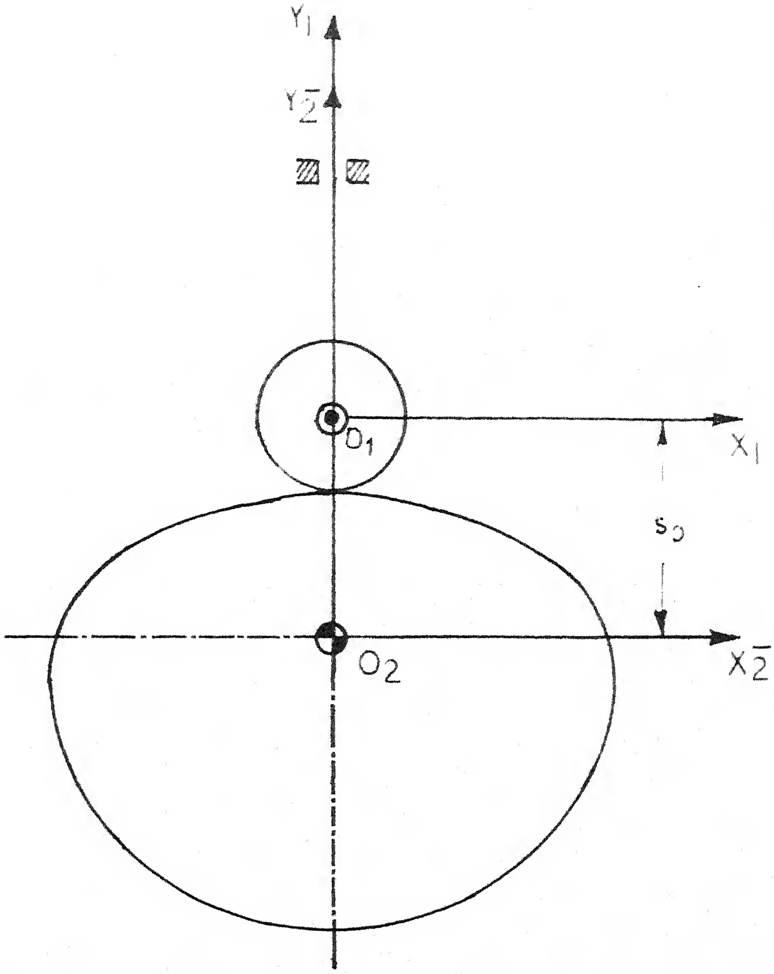


FIG. 3·5

CAM PROFILE

Table 3.3 Results of the Design of a Minimal Size Cam

1.	Roller radius	r	1.0	unit
2.	Offset distance	a	0.0	units
3.	Initial displacement of the follower	s_o	3.0	units
4.	Base radius	r_b	3.0	units
5.	Maximum value of the pressure angle	ψ_{max}	26.1093	degrees
6.	Minimum value of the radius of curvature	ρ_{min}	2.0	units

Example No.2

Requirements :

(i) a bed of a printing press has a reciprocating motion;

(ii) the printing cylinder rotates uniformly;

(iii) one cycle of bed motion is produced while the cylinder makes one revolution;

(iv) during the forward stroke, the bed has a period of uniform velocity, which is equal to the velocity on the periphery of the printing cylinder;

(v) the return stroke may be made during a shorter time;

(vi) the deceleration at the end of either stroke should be equal or close to the acceleration of the next stroke;

(vii) the bed of the press complates one cycle with a cam driving it rotates on 360^o [12].

Selection of the input-output motion curves and calculations for the period of the forward stroke and the return stroke are given in Appendix II. The input data is given in Table 3.4. Final results are presented in Table 3.5.

In this example roller radius is kept constant and equal to 2.0 units. Fig. 3.6 shows variations of the input-output motion parameters. Fig. 3.7 shows variation of θ . Here θ_{upper} and θ_{lower} gives the limits on variation of θ at which pressure angle is maximum. Fig. 3.8 shows variation of the pressure angle and the pressure angle constraint is critically satisfied, which is clearly seen in this figure. Fig. 3.9 gives variation of the radius of curvature and here the minimum value of the radius of curvature is well above the limiting value of the radius of curvature, which is seen in this figure. Cam profile is shown in Fig. 3.10.

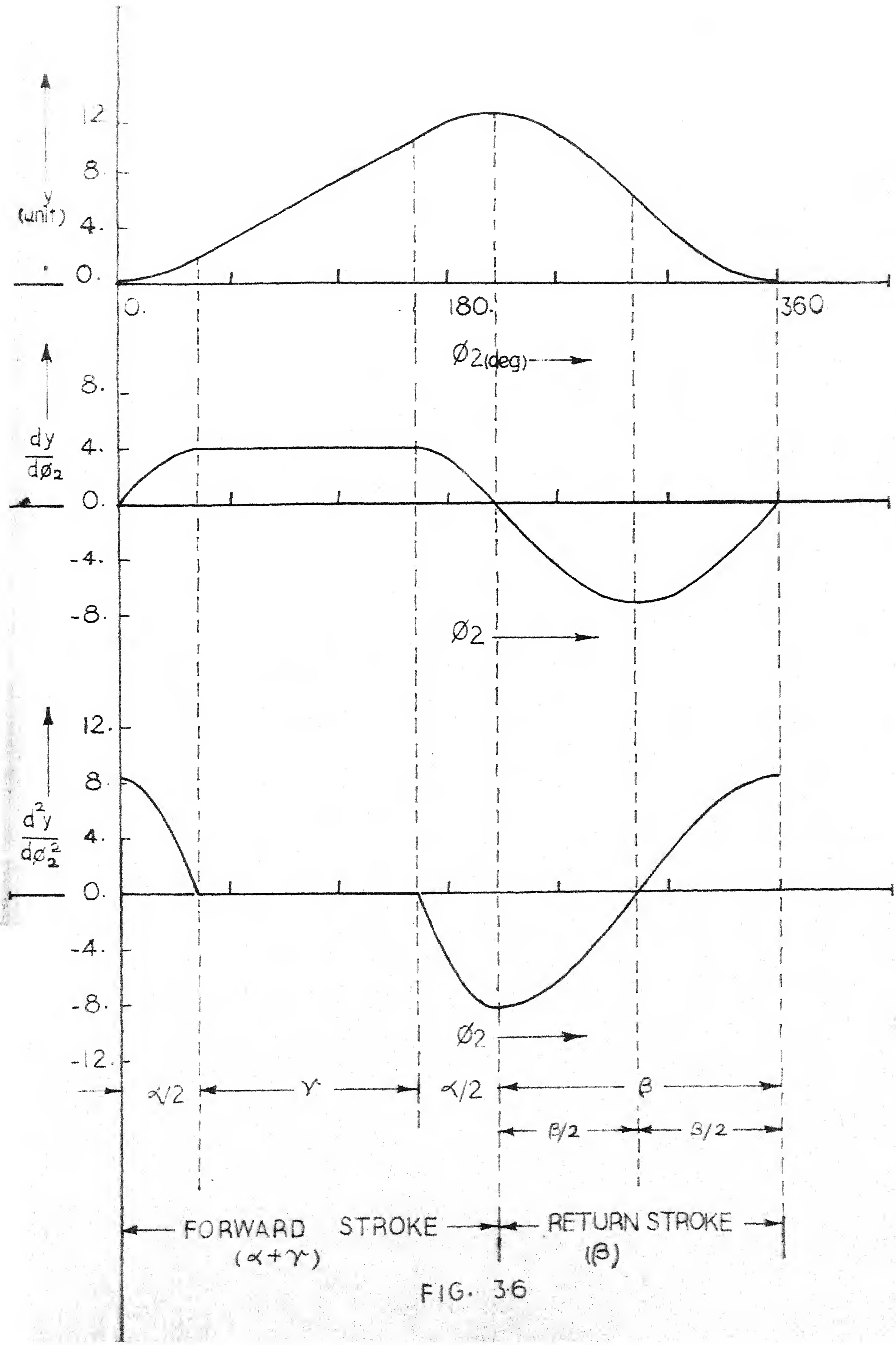
Table 3.4 Input Data of Example No.2

1.	Roller radius	r	2.0 units
2.	Limiting value of the maximum pressure angle	ψ^*	30.0°
3.	Limiting value of the minimum radius of curvature	ρ^*	4.0 units
4.	Kinematic Data :		

Range of ϕ_2 (in degree)	Range of s_1 units	Total Dis Displacement units	Input-Output Motion Curves
0.0 - 43.082	0.0 - 1.915	1.915	Half harmonic
43.082 - 163.082	1.915 - 10.292	8.378	Uniform velocity
163.082 - 206.164	10.292 - 12.207	1.915	Half harmonic
206.164 - 283.082	12.207 - 6.104	-6.104	Half harmonic
283.082 - 360.000	6.104 - 0.0	-6.104	Half harmonic

Table 3.5 Output

1.	Roller radius	r	2.0 units
2.	Offset distance	a	-0.7 units
3.	Initial displacement of the follower	s_o	6.5 units
4.	Base radius	r_b	6.538 units
5.	Maximum value of the pressure angle	ψ_{max}	29.955°
6.	Minimum value of the radius of curvature	ρ_{min}	6.252 units



y (unit)
12
8
4
0.
0.
180.
360
$\phi_{2(deg)}$
$\frac{dy}{d\phi_2}$
8.
4.
0.
-4.
-8.
ϕ_2
$\frac{d^2y}{d\phi_2^2}$
12.
8
4.
0.
-4.
-8.
-12.
ϕ_2
$\alpha/2$
γ
$\alpha/2$
β
$\beta/2$
$\beta/2$
FORWARD STROKE
$(\alpha+\gamma)$
RETURN STROKE
(β)

FIG. 36

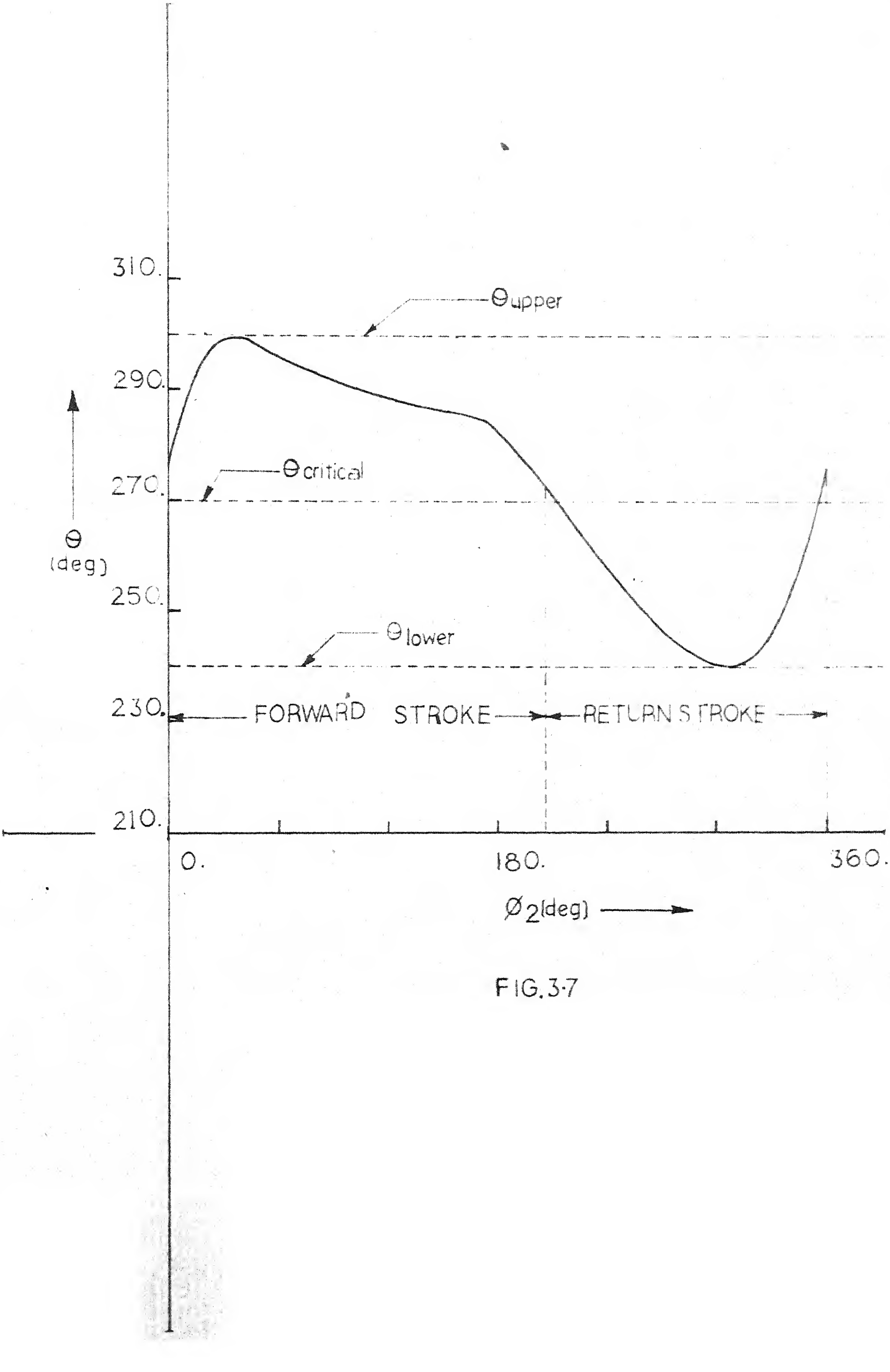

310.
290.
270.
250.
230.
210.
θ (deg)
θupper
θcritical
θlower
FORWARD STROKE
RETURN STROKE
0.
180.
360.
Ø2(deg)

FIG. 3·7

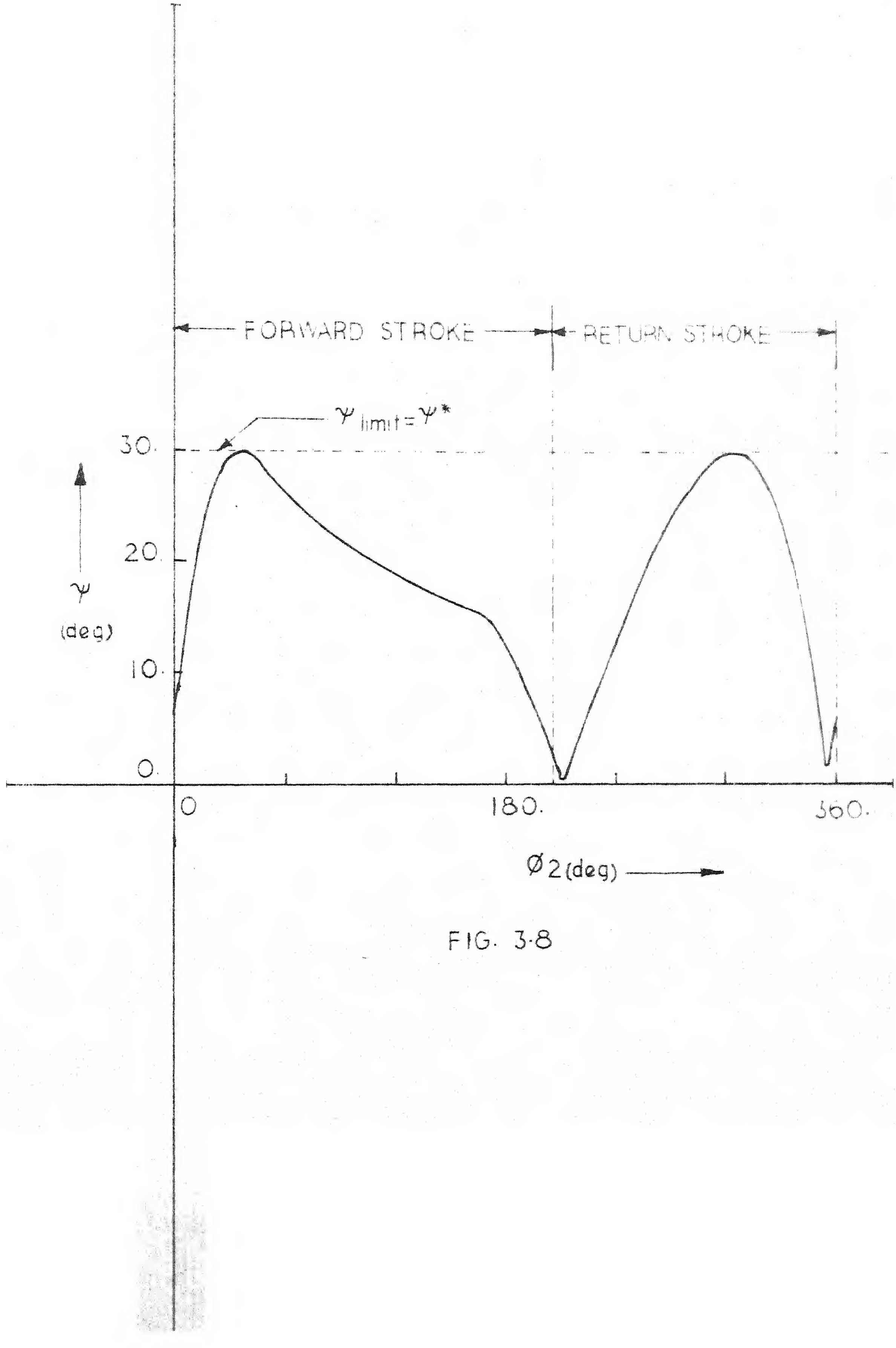

FORWARD STROKE
RETURN STROKE
$\psi_{limit} = \psi^*$
30.
20.
10.
0.
ψ
(deg)
0
180.
360.
ϕ_2 (deg)

FIG. 3·8

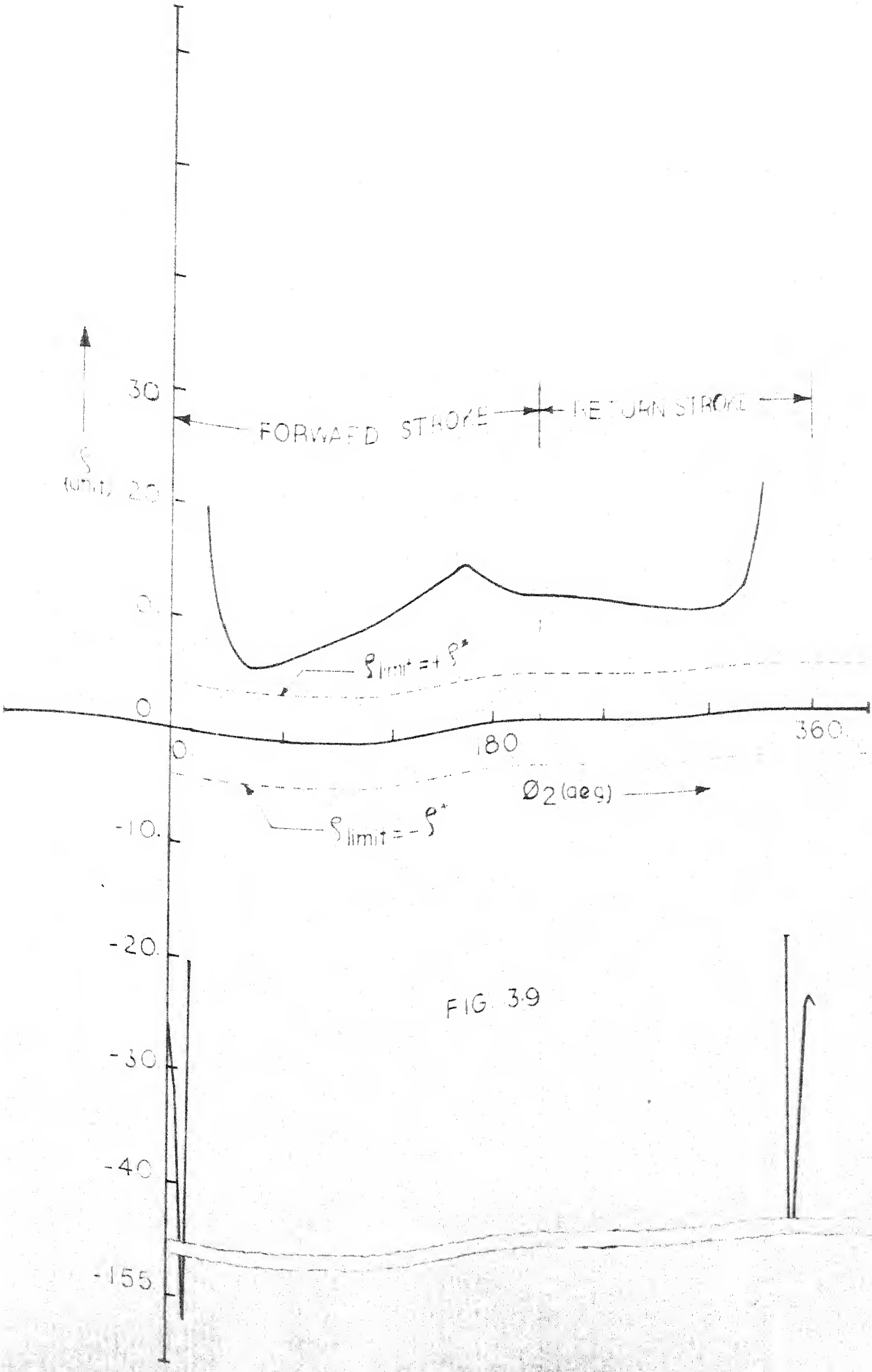

30
FORWARD STROKE
RETURN STROKE
ρ (unit)
20
10.
$\rho_{limit} = +\rho^*$
0
0.
180
360.
$\varnothing_2$ (deg)
-10.
$\rho_{limit} = -\rho^*$
-20.
-30.
-40
-155.

FIG. 3.9

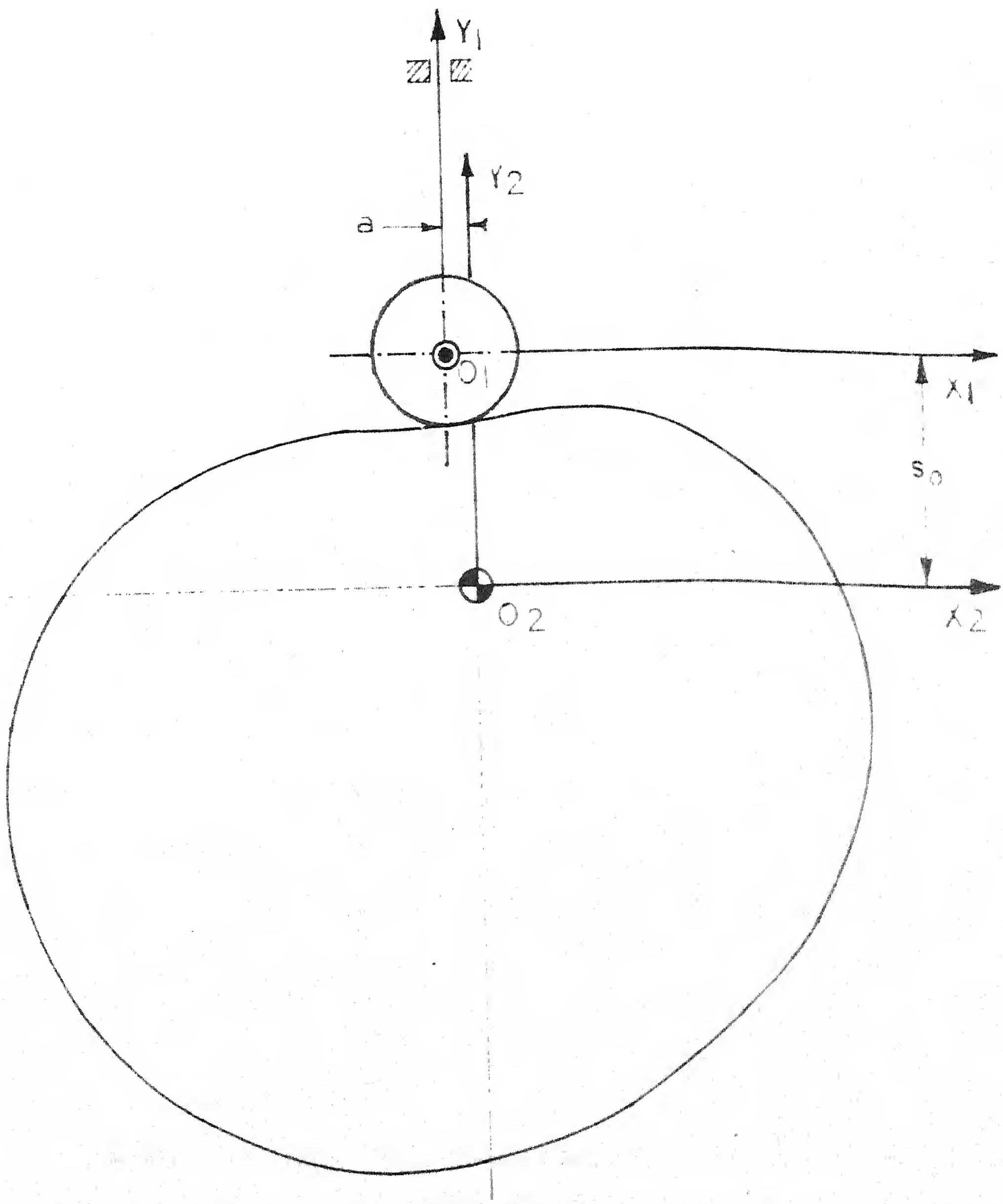


FIG. 3.10

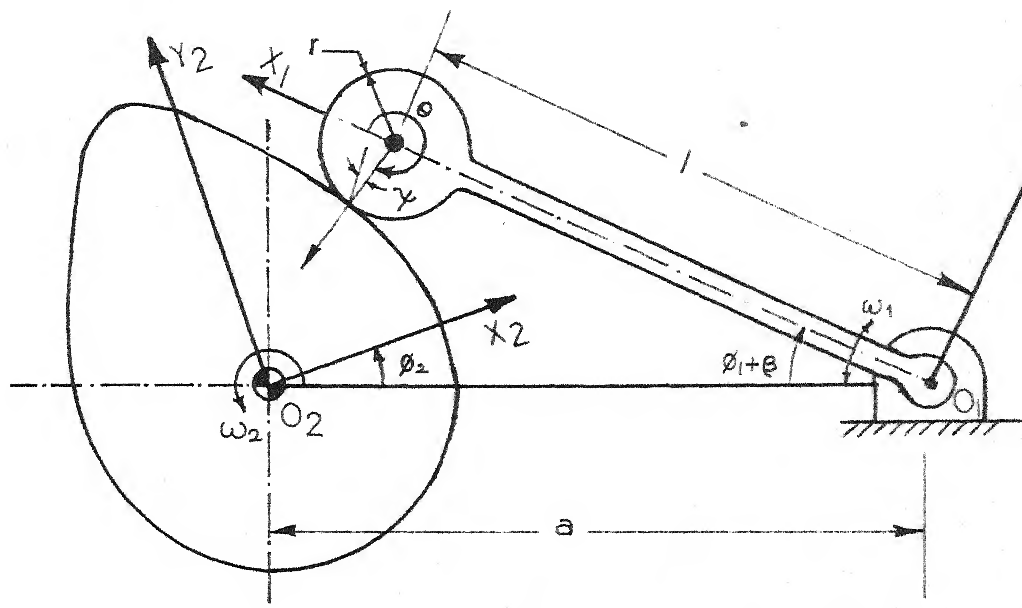
CAM PROFILE

Chapter 4

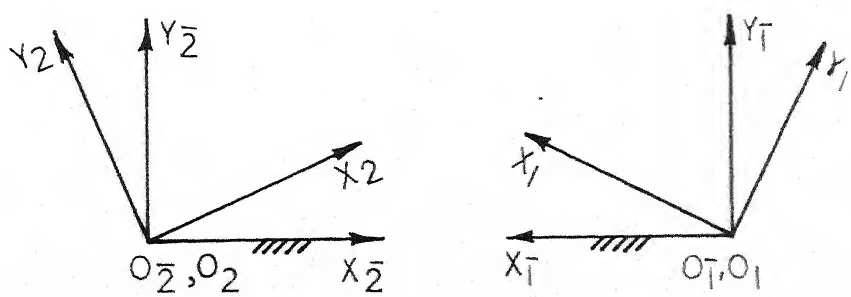
DESIGN OF A DISC CAM WITH AN OSCILLATING, ROLLER-ENDED FOLLOWER

4.1 Design Procedure

A design procedure for a disc cam with an oscillating roller-ended follower is discussed in this section. Both the pressure angle constraint as well as the radius of curvature constraints are taken into account. Fig. 4.1a shows a schematic diagram under discussion. Fig. 4.1b shows the orientation of various co-ordinate systems required in the analysis. The co-ordinate system $S_1(X_1, O_1, Y_1)$ is attached to the follower and the co-ordinate system $S_2(X_2, O_2, Y_2)$ is attached to the cam. Co-ordinate systems $S_{\bar{1}}(X_{\bar{1}}, O_{\bar{1}}, Y_{\bar{1}})$ and $S_{\bar{2}}(X_{\bar{2}}, O_{\bar{2}}, Y_{\bar{2}})$ give the initial positions of S_1 and S_2 respectively. A global co-ordinate system, fixed in space, is designated by $S(X,O,Y)$ and, in the present case, $S_{\bar{1}} \equiv S$. The cam rotates with a uniform angular velocity w_2. The input parameter of motion of the cam is the angular rotation of the cam, ϕ_2. The angular rotation of the follower from its initial position is the output parameter of the motion and is denoted by ϕ_1. The distance 'a' denotes the distance between the pivot of the cam and the pivot of the follower arm. The initial position of the follower is given by the angle β. At any instant, given by the angular rotation of the cam ϕ_2,



SCHEMATIC DIAGRAM
FIG.4.1a



COORDINATE SYSTEMS
FIG.4.1b

P denotes the point of contact between the follower surface and the cam surface. The angular rotation of the follower ϕ_1 is a function of ϕ_2 and is given by

$$\phi_1 = \phi_1(\phi_2) \tag{4.1}$$

The unit normal vector $\underline{n}^{(P)}$ at the point P on the follower surface is given by [10],

$$\underline{n}^{(P)} = |\, C(\theta + \phi_1),\ S(\theta + \phi_1), 0\,|^T \tag{4.2}$$

where θ is measured from X axis and denotes the position of contact at the given instant. The parametric equations for the follower roller surface, the cam profile, the pressure angle, the radius of curvature and the condition of contact are derived in [10] and are as follows :

(i) The follower roller surfaee

$$\underline{R}^{(P)} = \begin{vmatrix} lC\phi_1 + rC(\theta + \phi_1) \\ lS\phi_1 + rS(\theta + \phi_1) \\ 1 \end{vmatrix} \tag{4.3}$$

where r is the radius of the roller and l is the arm length of the follower.

(ii) The cam profile

$$\underline{R}_2^{(P)} = |x_2, y_2, 1|^T \tag{4.4}$$

$$x_2 = -lC(\phi_1+\phi_2+\beta) - rC(\theta+\phi_1+\phi_2+\beta) + aC\phi_2$$

$$y_2 = lS(\phi_1+\phi_2+\beta) + rS(\theta+\phi_1+\phi_2+\beta) - aS\phi_2$$

(iii) The condition of contact, θ

$$\theta = \tan^{-1}\left[\frac{aS(\phi_1+\beta)}{l(1+\phi_1') - aC(\phi_1+\beta)}\right] \tag{4.5}$$

(iv) The pressure angle, ψ

$$\psi = \cos^{-1}\left[\left|\frac{lS\theta}{(l^2+r^2+2lrC\theta)^{\frac{1}{2}}}\right|\right] \tag{4.6}$$

(v) The radius of curvature, ρ

$$\rho = \frac{1}{\chi} \tag{4.7}$$

where χ is curvature and

$$\chi = -(1 + k_4/k_5)/r$$

$$k_4 = ((1+\phi_1')\ lC - aC(\theta+\phi_1+\beta))^2$$

$$k_5 = r(\phi_1''\ lS\theta - a\phi_1'\ C(\theta+\phi_1+\beta)) - ((1+\phi_1')lC\theta -$$

$$aC(\theta+\phi_1+\beta))\ ((1+\phi_1')(bC\theta+r) - aC(\theta+\phi_1+\beta))$$

General design procedure is discussed in Section 2.3 of Chapter 2. For the roller-ended oscillating follower case the design variables are,

i) the roller radius, r,

ii) the base radius, r_b,

iii) the distance between the pivot of the cam and the pivot of the follower arm, a,

iv) the follower arm length, l and

v) the angle giving the initial position of the follower, β.

Some of the above mentioned variables are modified into non-dimensional variables for computational purpose and are given as followes :

i) $\eta = r/l$,

ii) $\lambda = r_b/l$ and

iii) $\zeta = a/l$.

The design procedure, given in the form of a flow chart in Fig. 4.2, is as follows.

Step 1 : Choose input-output motion curves according to the requirements of the problem. Read the values of η_{lower}, η_{upper}, step η, β_{lower}, β_{upper}, step β, ψ^*, ρ^*, ε_1.

Step 2 : Assign the value of $\eta = \eta_{lower}$.

Step 3 : Assign the value of $\eta = \eta$ + step η and $\beta = \beta_{lower}$.

Step 4 : Compare the values of η and η_{upper}. If η is greater than η_{upper} go to Step 17.

Step 5 : Find the values of θ_{lower} and θ_{upper}.

Step 6 : Assign the value of $\beta = \beta$ + step β.

Step 7 : Compare the values of β and β_{upper}. If β is greater than β_{upper} go to Step 3.

Step 8 : Select the value of ζ . Find the value of θ_{max}.

Step 9 : Compare the values of θ_{max} and θ_{upper}. If θ_{max} is greater than θ_{upper} to go to Step 6.

Step 10 : Find the value of θ_{min}.

Step 11 : Compare the values of θ_{min} and θ_{lower}. If θ_{min} is less than θ_{lower} go to Step 6.

Step 12 : Find the value of ρ_{min}.

Step 13 : Compare the values of ρ_{min} and ρ^*. If ρ_{min} is less than ρ^* go to Step 6.

Step 14 : Compare the values of step β and ε_1. If step β is less than or equal to ε_1 go to Step 16.

Step 15 : Assign the value of $\beta = \beta$ - step β and step β = step $\beta/2$. Go to Step 6.

Step 16 : Find the size of cam and go to Step 3.

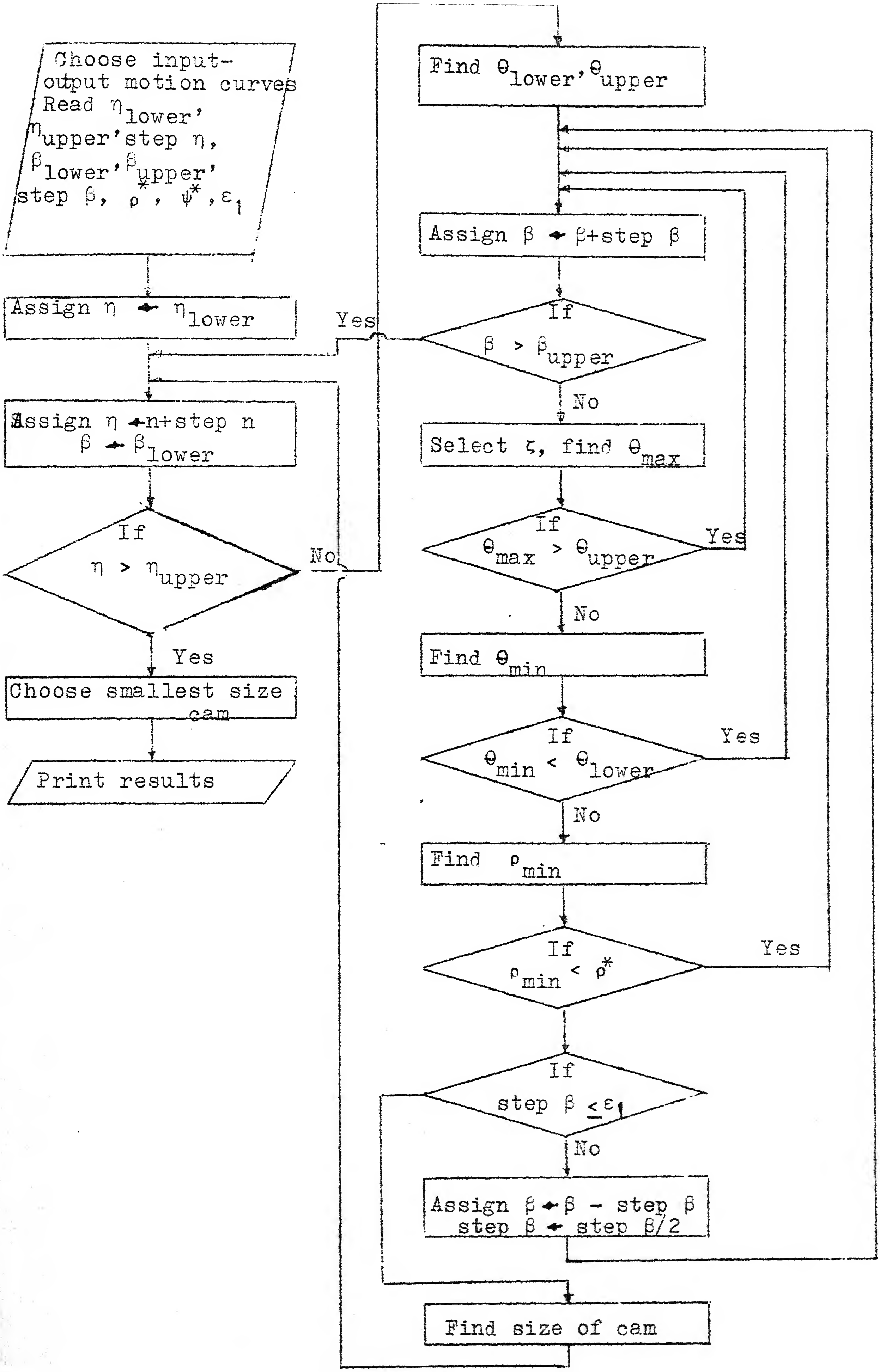
Step 17 : Compare the different sizes of cam and choose the smallest size cam.

Step 18 : Print the results of this smallest size cam.

4.2 Numerical Examples

Example No.1

This example illustrates the design procedure applied to a disc cam with an oscillating roller-ended follower. The input data is given in Table 4.1. The output is presented in



Choose input-output motion curves
Read η lower, η upper, step η, β lower, β upper, step β, ρ*, ψ*, ε1
Assign η ← η lower
Assign η ← η+step η
β ← β lower
If η > η upper
No
Yes
Choose smallest size cam
Print results
Find θ lower, θ upper
Assign β ← β+step β
If β > β upper
Yes
No
Select ζ, find θ max
If θ max > θ upper
Yes
No
Find θ min
If θ min < θ lower
Yes
No
Find ρ min
If ρ min < ρ*
Yes
If step β ≤ ε1
No
Assign β ← β − step β
step β ← step β/2
Find size of cam

Table 4.2. Comparing different values of λ^s for different cases of η, we select that value of η which gives smallest value of λ. Then applying again the same search technique we search in the neighbourhood of this value of η, which gives exact (to certain extent) lowest value of λ and the corresponding value of η. Taking this value of η as final design we find all other variables. Final results are given in Table 4.3.

Fig. 4.3 shows the variation of the pressure angle, ψ. Fig. 4.4 shows the variation of the radius of curvature, ρ. Cam profile is shown in Fig. 4.5.

Fig. 4.3 shows that the pressure angle constraint is not critically satisfied but the radius of curvature constraint is critically satisfied which is seen clearly in Fig. 4.4. Fig. 4.5 shows the cam profile.

Example No.2

Requirements :

i) a cam follower mechanism is to be designed which is used in calculating equipment to pick off a punched card from the stack.

ii) provide initial acceleration to achieve certain peripheral velocity to the feed rolls,

iii) picker knife should proceed with constant velocity until the feed rolls pick up the card,

Table 4.1 Input Data of Example No.1

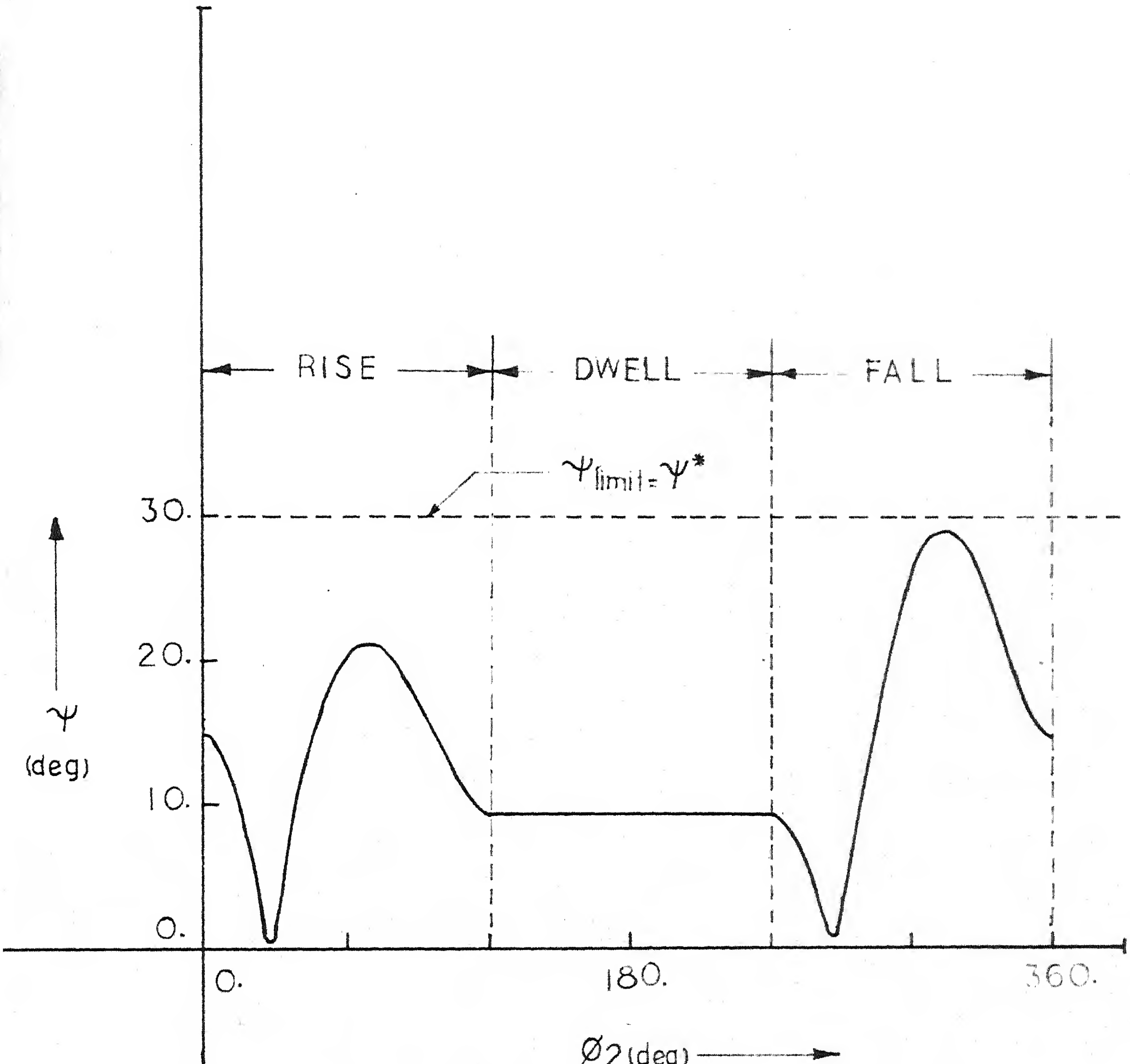
1.	Roller radius	r	1.00	unit
2.	Lower value of η	η_{lower}	0.1	
3.	Upper value of η	η_{upper}	0.4	
4.	Lower value of β	β_{lower}	0.0	degree
5.	Upper value of β	β_{upper}	65.0	degrees
6.	Limiting value of the maximum pressure angle	ψ^*	30.0	degrees
7.	Limiting value of the minimum radius of curvature	ρ^*	2.0	units
8.	Kinematic Data :			

Range of ϕ_2 degree	Range of ϕ_1 degree	Total Displacement (degree)	Input-Output Motion Curve
0 - 180	0 - 30	30.00	Full cycloid
180 - 240	30 - 30	0.0	-
240 - 360	30 - 0	-30.0	Full cycloid

Note: Detailed equations for input-output motion curves are given in Appendix I.

Table 4.2 Results of the Design Scheme for Example No.1

η	λ
0.10	0.9214
0.20	0.7451
0.30	0.7103
0.40	0.9653



RISE
DWELL
FALL
$\psi_{limit} = \psi^*$
30.
20.
10.
0.
ψ (deg)
0.
180.
360.
ϕ_2 (deg)

FIG. 4.3

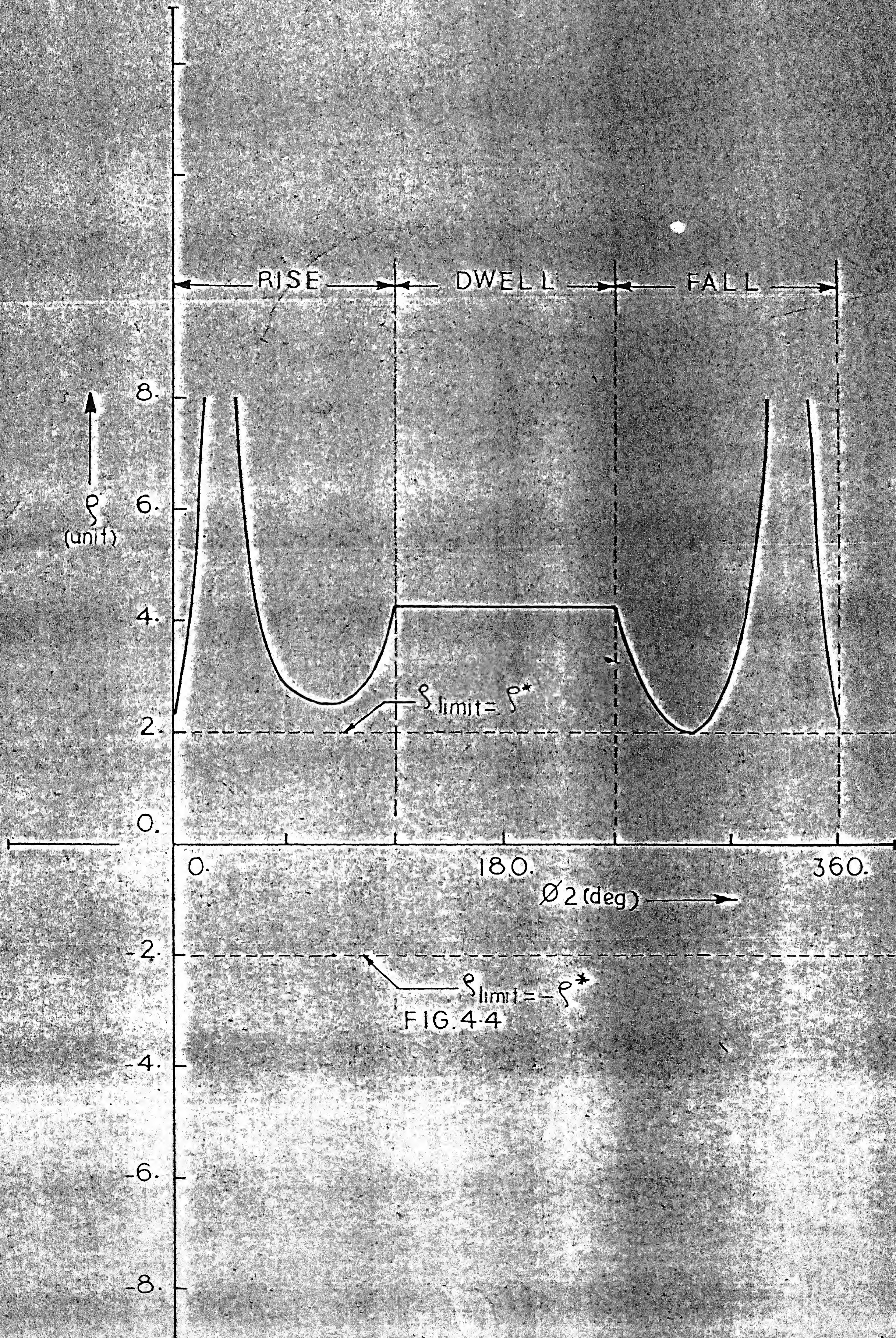

RISE
DWELL
FALL
8.
6.
4.
2.
0.
-2.
-4.
-6.
-8.
ρ (unit)
$\rho_{limit} = \rho^*$
$\rho_{limit} = -\rho^*$
0.
180.
360.
$\varnothing 2$ (deg)

FIG. 4.4

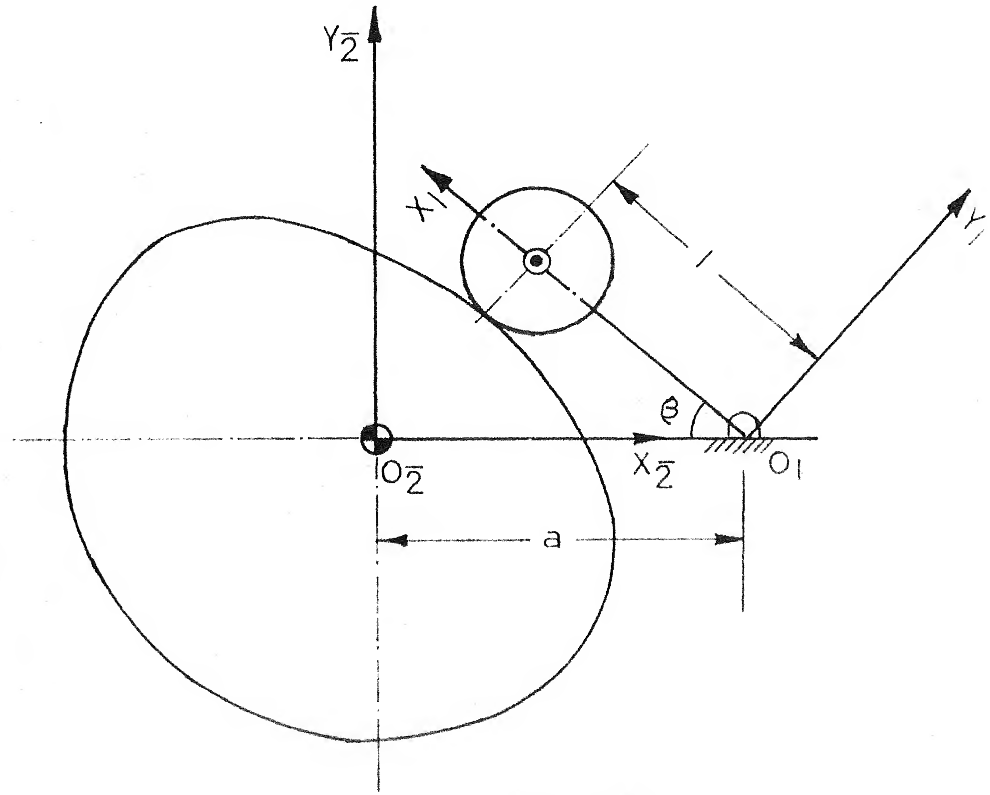


FIG 4.5

CAM PROFILE

Table 4.3 Results of the Design of a Minimal Size Cam

1.	Roller radius	r	1.00	unit
2.	Angle giving the initial position of follower	β	41.0547	degrees
3.	Distance between the pivot of the cam and the pivot of follower arm	a	5.0570	units
4.	Follower arm length	l	3.8095	units
5.	Base radius	r_b	2.3214	units
6.	Maximum value of the pressure angle	ψ_{max}	29.7991	degrees
7.	Minimum value of the radius of curvature	ρ_{min}	2.00	units

iv) picker knife then decelerates and returns to the starting point,

v) return stroke should be of shorter duration [5].

So essentially one complete rotation of the cam is divided in the following sections.

Dwell - Rise (acceleration) - Rise (constant velocity) - Rise (deceleration) - Fall - Dwell.

Because the follower does not move radially but swing in an arc, the movement of the picker will not be exactly what cam provides. To reduce the distorsions introduced in the actual motion curves, provide follower arm length maximum possible.

So design starts with constant follower arm length rather than constant roller radius.

Selection of the input-output motion curves and the calculations for the period of rise and fall are given in Appendix II. The input data is given in Table 4.4. Final results are presented in Table 4.5. Fig. 4.6 shows variation of the input-output motion parameters. Fig. 4.7 shows variation of θ, θ_{lower} and θ_{upper} being limiting values of θ at which the pressure angle is maximum. Fig. 4.8 shows variation of the pressure angle. Fig. 4.9 shows variation of the radius of curvature. Cam profile is shown in Fig.4.10.

The pressure angle constraint is critically satisfied which is seen in Fig. 4.8, whereas the minimum radius of curvature is well within the limit which is seen in Fig.4.9 clearly.

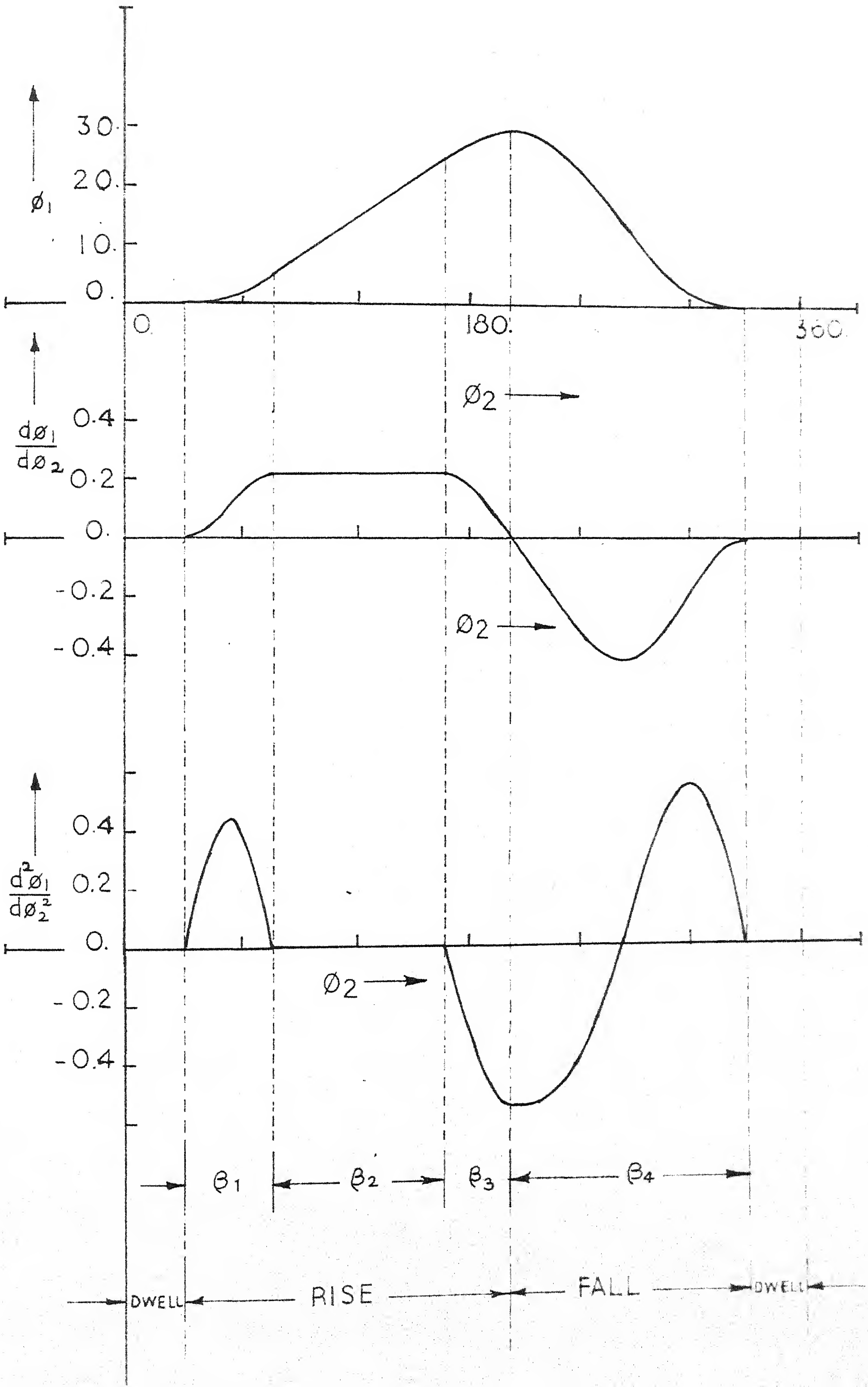
Table 4.4 Input Data of Example No. 2

1.	Follower arm length	l	10.0	units
2.	Lower value of η	η_{lower}	0.10	-
3.	Upper value of η	η_{upper}	0.50	-
4.	Lower value of β	β_{lower}	0.0	degree
5.	Upper value of β	β_{upper}	65.0	degrees
6.	Limiting value of the maximum pressure angle	ψ^*	30.0	degrees
7.	Limiting value of the minimum radius of curvature	ρ^*	2xroller radius	units
8.	Kinematic Data :			

Range of ϕ_2 degree	Range of ϕ_1 degree	Total Displacement (degree)	Input-Output Motion Curve
0 - 30.0	0 - 0.0	0.0	-
30 - 75.478	0 - 5.0	5.0	Half cycloid
75.478-166.435	5 - 25.0	20.0	Uniform velocity
166.435-202.154	25 - 30.0	5.0	Half harmonic
202.154-330.0	30 - 0.0	-30.0	Polynomial
330.000-360.0	0 - 0.0	0.0	-

Table 4.5 Output

1.	Roller radius	r	1.0	unit
2.	Angle giving the initial position of follower	β	23.6895	degrees
3.	Distance between the pivot of cam and the pivot of follower arm	a	10.4620	units
4.	Follower arm length	l	10.00	units
5.	Base radius	r_b	3.2243	units
6.	Maximum value of the pressure angle	ψ_{max}	29.8443	degrees
7.	Minimum value of the radius of curvature	ρ_{min}	3.2243	units


30.
20.
10.
0.
ϕ_1
0.
180.
360.
ϕ_2
0.4
0.2
0.
-0.2
-0.4
$\frac{d\phi_1}{d\phi_2}$
ϕ_2
0.4
0.2
0.
-0.2
-0.4
$\frac{d^2\phi_1}{d\phi_2^2}$
ϕ_2
β_1
β_2
β_3
β_4
DWELL
RISE
FALL
DWELL

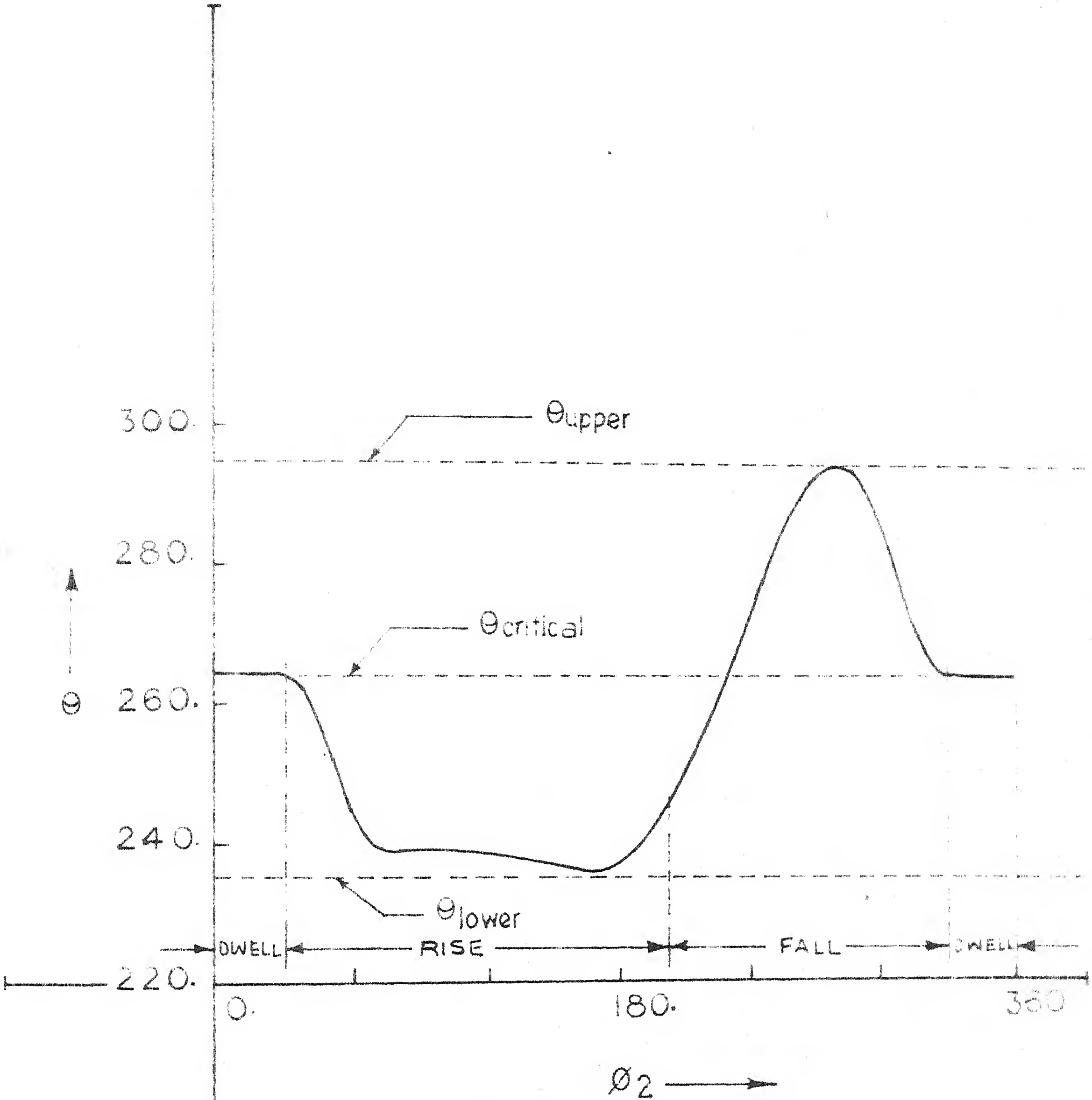

300.
280.
260.
240.
220.
θ
θupper
θcritical
θlower
DWELL
RISE
FALL
DWELL
0.
180.
360
Ø2

FIG. 4·7

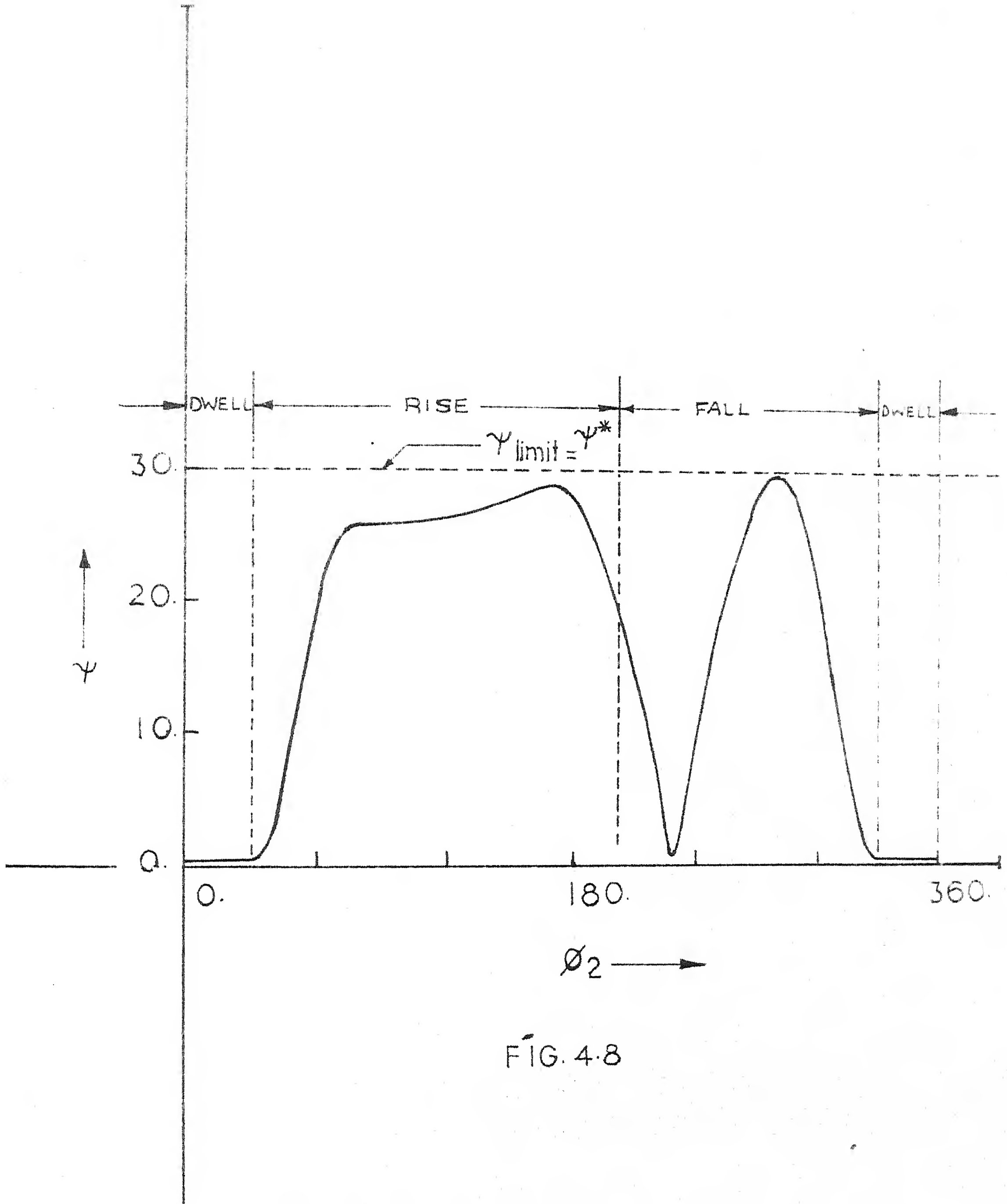

DWELL
RISE
FALL
DWELL
$\psi_{limit} = \psi^*$
30.
20.
10.
0.
ψ
0.
180.
360.
ϕ_2

FIG. 4.8

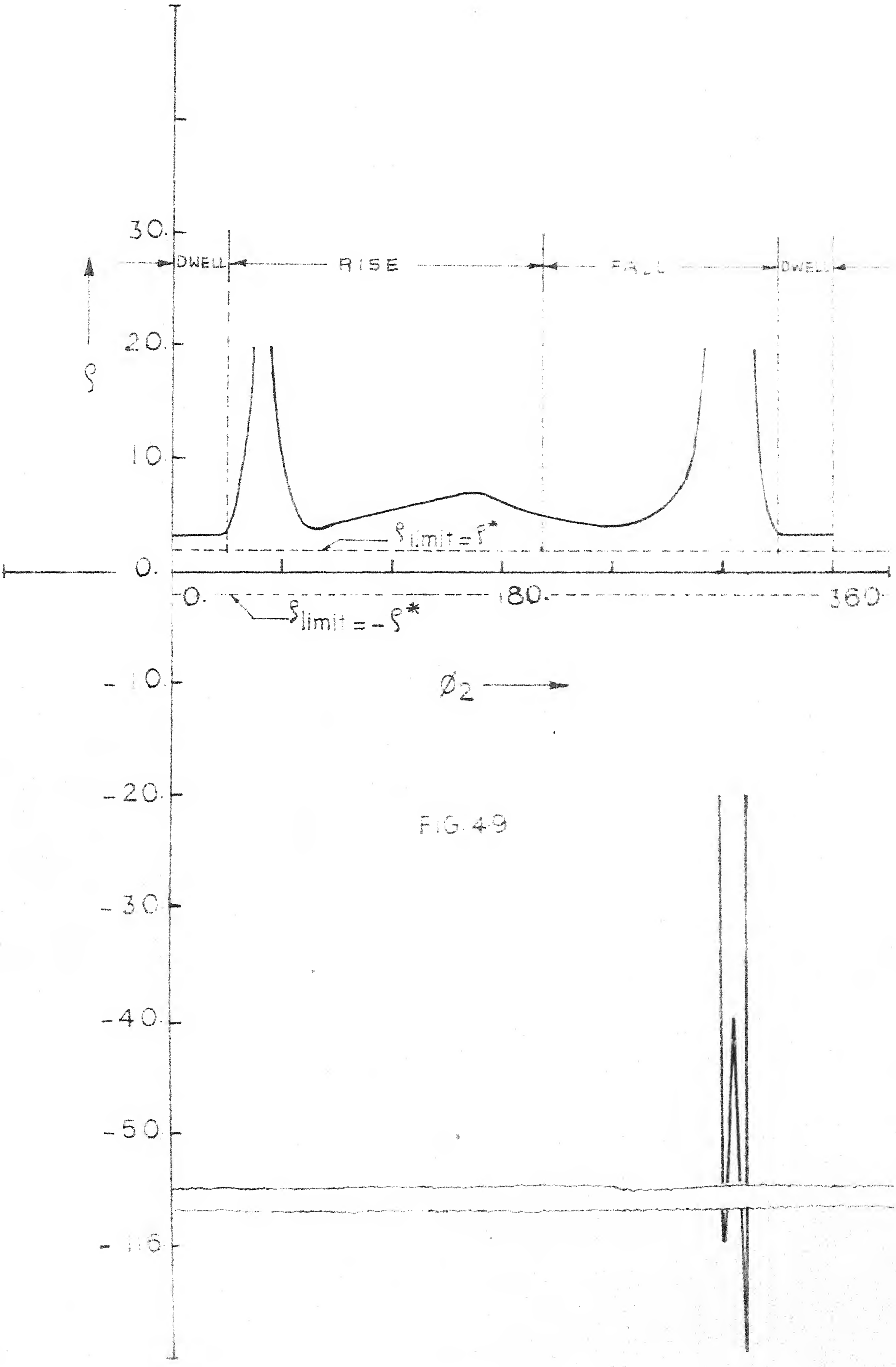

30.
20.
10.
0.
-10.
-20.
-30.
-40.
-50.
-115
DWELL
RISE
FALL
DWELL
ρ
$\rho_{limit} = \rho^*$
0.
180.
360
$\rho_{limit} = -\rho^*$
ϕ_2

FIG 4.9

Chapter 5

SUMMARY AND CONCLUSIONS

In the present work an approach for the design of disc cam mechanisms based on optimal operational requirements has been developed. Two important aspects of operation are that the maximum value of the pressure angle and the minimum value of the radius of curvature of the cam profile be within prescribed limits. The proposed approach is based on certain numerical techniques. On the other hand prevalent procedure for such design problems are essentially based on graphical constructions [4]. The drawbacks using these graphical procedure can be listed as follows.

i) The nomograms prepared provide the values of the parameters such as the maximum pressure angle or the minimum radius of curvature for a given set of variables. One has to select these design variables arbitrarily. If the values of the maximum pressure angle and the minimum radius of curvature are not satisfactory then a new set of values of the design variables is selected. There is no systematic procedure to find the new set of design variables. In short the prevalent procedure is more of a trial-and-error type.

ii) For the mechanisms used at higher speeds, finding the accurate values of the design variables is more important since small errors in the value of design variables will produce additional accelerations which will adversely affect the dynamic properties of the mechanism.

iii) At higher speeds weight or size of the mechanism also plays an important role from the point of view of increased vibrations. So, along with satisfying the above mentioned two constraints, searching for a minimal size cam is also equally important, which is not possible with graphical constructions.

The proposed scheme of design is based on the analytical expressions for the parameters - the pressure angle and the radius of curvature. Moreover, it is necessary to evaluate the rotational position of the cam at which the pressure angle is maximum and the magnitude of the pressure angle at this position. Similarly, it is necessary to evaluate the minimum value of the radius of curvature and the corresponding position of the cam rotation.

Evaluation of the above mentioned quantities is a difficult task for cam mechanisms. This is because of the highly nonlinear and parametric nature of the analytical expressions of the pressure angle and the radius of curvature. It is because of these reasons that a numerical schemes has been proposed in this work. The accuracy of the numerical scheme is satisfactory.

Following are the salient features of the procedures developed in the present work.

i) The general procedure is applied to the design of a disc cam with both, a translating roller-ended follower and an oscillating roller-ended follower with very little modifications.

ii) The procedure developed is general in nature. It gives flexibility to the designer to put the limits on the values of the constraints as well as one can very the range of the design variables according to the operational requirements.

iii) The design procedure has been illustrated by means of an example of a cam driving the bed of the printing press (a disc cam with the translating roller-ended follower). Another illustrative example is that of a popular mechanism used in calculating machinary (a disc cam with an oscillating roller-ended follower).

Following are the suggestions for the further work.

i) The present approach does not take into account the constraints related to the dynamic behaviour of the cam mechanisms. So, as to evolve a satisfactory design it will be necessary to take into account not only the constraints on the maximum pressure angle and minimum radius of curvature but also the dynamic characteristics of the cam mechanisms.

ii) The present work does not take into account the constraints on the tolerances on the cam profile. Mechanical errors in the form of inaccuracies of the cam profile produce undesirable noise and vibrations. It is therefore, desirable to develop the synthesis procedure that takes into account the constraints on such dynamic behaviour of cam mechanisms.

iii) The present work deals only with two dimensional cam-follower mechanisms. It is necessary to extend the present approach to three dimensional cam-follower mechanisms.

REFERENCES

[1] Hanson, R.S., and Churchill, F., Theory of Envelopes Provides New Design Equations, Product Engineering, Vol. 33, No.17, Aug. 20, 1962, pp. 45-55.

[2] Dhande, S.G., Bhadoria, B.S., and Chakraborty, J., A Unified Approach to the Analytical Design of Three Dimensional Cam Mechanisms, Journal of Engineering for Industry, Transactions ASME, Series B, Vol. 95, No.2, May, 1973, pp. 589-597.

[3] Rothbart, H.A., Cams - Design, Dynamics and Accuracy, John Wiley, New York, 1956.

[4] Jensen, P.W., Cam Design and Manufacture, Industrial Press, New York, 1965.

[5] Kloomak, M., and Muffley, R.V., 'Plate Cam Design - With Emphasis on Dynamic Effects', Product Engineering, Vol.26, No.2, 1955, pp. 156-162.

[6] Kloomak, M., and Muffley, R.V., 'Plate Cam Design - Pressure Angle Analysis', Product Engineering, Vol. 26, No.5, 1955, pp. 155-171.

[7] Kloomak, M., and Muffley, R.V., 'Plate Cam Design - Radius of Curvature', Product Engineering, Vol. 26, No.9, 1955, pp. 186-209.

[8] Ganter, M.A., Uicker, J.J., Design Charts for Disc Cams with Reciprocating Radial Roller Followers, Paper No.78-DET-36, Transactions ASME.

[9] Mischke, C., Optimal Offset on Translating Follower Plate Cams, Journal of Engineering for Industry, Transactions ASME, Feb. 1970, pp. 172-176.

[10] Chakraborty, J., and Dhande, S.G., Kinematics and Geometry of Planar and Spatial Cam Mechanisms, Wiley Eastern, New Delhi/Bangalore, 1977.

[11] Rao, S.S., Optimization-theory and applications, Wiley Eastern, New Delhi/Bangalore, 1978.

[12] Neklutin, C.N., Mechanisms and Cams for Automatic Machines, American Elsevier, New York, 1969.

Appendix I

INPUT-OUTPUT MOTION CURVES

The detailed equations for the input-output motion curves are presented by [8]. Here the detailed equations of only those curves which are selected for numerical examples in Chapter 3 and Chapter 4 are given.

1. Full cycloid lift curve (C10)

$$y = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin 2\pi\frac{\theta}{\beta}\right)^*$$

$$y' = \frac{L}{\beta}\left(1 - \cos 2\pi\frac{\theta}{\beta}\right)$$

$$y'' = \frac{2\pi L}{\beta^2}\left(\sin 2\pi\frac{\theta}{\beta}\right)$$

2. Full cycloid lift curve (C20)

$$y = L\left(1 - \frac{\theta}{\beta} + \frac{1}{2\pi}\sin 2\pi\frac{\theta}{\beta}\right)$$

$$y' = -\frac{L}{\beta}\left(1 - \cos 2\pi\frac{\theta}{\beta}\right)$$

$$y'' = -\frac{2\pi L}{\beta^2}\left(\sin 2\pi\frac{\theta}{\beta}\right)$$

3. Half cycloid lift curve (C11)

$$y = L\left(\frac{\theta}{\beta} - \frac{1}{\pi}\sin \pi\frac{\theta}{\beta}\right)$$

$$y' = \frac{L}{\beta}\left(1 - \cos \pi\frac{\theta}{\beta}\right)$$

$$y'' = \frac{\pi L}{\beta^2}\left(\sin \pi\frac{\theta}{\beta}\right)$$

4. Eight-order polynomial curve (820)

$$y = L[1.00000 - 2.63415\left(\frac{\theta}{\beta}\right)^2 + 2.78055\left(\frac{\theta}{\beta}\right)^5 + 3.17060\left(\frac{\theta}{\beta}\right)^6 - 6.87795\left(\frac{\theta}{\beta}\right)^7 + 2.56095\left(\frac{\theta}{\beta}\right)^8]$$

$$y' = \frac{L}{\beta}[-5.26830\left(\frac{\theta}{\beta}\right) + 13.90275\left(\frac{\theta}{\beta}\right)^4 + 19.02360\left(\frac{\theta}{\beta}\right)^5 - 48.14565\left(\frac{\theta}{\beta}\right)^6 + 20.48760\left(\frac{\theta}{\beta}\right)^7]$$

$$y'' = \frac{L}{\beta^2}[-5.26830 + 55.61100\left(\frac{\theta}{\beta}\right)^3 + 95.11800\left(\frac{\theta}{\beta}\right)^4 - 288.87390\left(\frac{\theta}{\beta}\right)^5 + 143.41320\left(\frac{\theta}{\beta}\right)^6]$$

5. Half harmonic lift curve (H11)

$$y = L(1 - \cos\frac{\pi\theta}{2\beta})$$

$$y' = \frac{\pi L}{2\beta}(\sin\frac{\pi\theta}{2\beta})$$

$$y'' = \frac{\pi^2 L}{4\beta^2}(\cos\frac{\pi\theta}{2\beta})$$

6. Half harmonic lift curve (H21)

$$y = L(\cos\frac{\pi\theta}{2\beta})$$

$$y' = -\frac{\pi L}{2\beta}(\sin\frac{\pi\theta}{2\beta})$$

$$y'' = -\frac{\pi^2 L}{4\beta^2}(\cos\frac{\pi\theta}{2\beta})$$

7. Half harmonic lift curve (H12)

$$y = L(\sin \frac{\pi\theta}{2\beta})$$

$$y' = \frac{\pi L}{2\beta}(\cos \frac{\pi\theta}{2\beta})$$

$$y'' = -\frac{\pi^2 L}{4\beta^2}(\sin \frac{\pi\theta}{2\beta})$$

8. Half harmonic lift curve (H22)

$$y = L(1 - \sin \frac{\pi\theta}{2\beta})$$

$$y' = -\frac{\pi L}{2\beta}(\cos \frac{\pi\theta}{2\beta})$$

$$y'' = \frac{\pi^2 L}{4\beta^2}(\sin \frac{\pi\theta}{2\beta})$$

---

* θ - input motion parameters, cam rotation

y - output motion parameter, follower displacement

y' - $dy/d\theta$

y'' - $d^2y/d\theta^2$

L - total displacement of follower

β - range of cam rotation

Appendix II

SELECTION OF INPUT-OUTPUT MOTION CURVES

Selection of input-output motion curves is done depending on the requirement of the problem. After selecting these curves the calculation required to find the magnitudes of different periods of cam rotation are discussed here.

<u>For Example No.2 of Section 3.2</u> :

In this example design of a disc cam with a translating roller-ended follower mechanism is discussed.

Nomenclature used in this example is as follows :

α^o - period of acceleration and deceleration during the forward stroke

v^o - period of the uniform velocity during the forward stroke

β^o - period of the return stroke

S - displacement during the varying velocity period of the forward stroke

f - displacement during the uniform velocity period

$F = f + S$ total stroke

V_f - maximum velocity during the forward stroke

V_2 - maximum velocity during the return stroke

A_f - maximum acceleration or deceleration during the forward stroke

A_r - the same during the return stroke

D - the diameter of printing cylinder

N - speed of the press or the number of cycles per second.

Input-output motion curves selected are as follows :

(i) Rise (acceleration) - Half harmonic lift curve (H11)

(ii) Rise (uniform velocity) - Straight line lift curve

(iii) Rise (deceleration) - Half harmonic lift curve (H12)

(iv) Fall (acceleration) - Half harmonic lift curve (H21)

(v) Fall (deceleration) - Half harmonic lift curve (H22)

The velocity at the periphery of the printing cylinder is πDN;

$$V_f = \pi DN = \frac{\pi}{2} \times \frac{S/2}{\frac{\alpha}{2} \cdot \frac{\pi}{180}} \; w \; ; \quad \text{where} \quad w = 2\pi N$$

therefore,

$$S = D \cdot \frac{\alpha}{180} \qquad (II.1)$$

During the period of uniform velocity, the following ratios are equal

$$\frac{v^o}{360} = \frac{f}{\pi D}$$

therefore,

$$v^o = \frac{f}{\pi D} \cdot 360 \qquad (II.2)$$

Now,

$$\alpha^o + \beta^o = 360 - v^o$$

therefore,

$$\beta^o = 360 \times \frac{\pi D - f}{\pi D} - \alpha^o \qquad (II.3)$$

The maximum acceleration and the maximum deceleration of both the strokes should be equal :

$$A_f = A_r \quad \text{or} \quad w^2 \times \frac{\pi^2}{4} \times \frac{S/2}{(\alpha/2 \times \pi/180)^2} = w^2 \times . \frac{\pi^2}{4} \times \frac{(S+f)/2}{(\beta/2 \cdot \pi/180)^2}$$

or

$$\frac{S}{\alpha^2} = \frac{S+f}{\beta^2} \tag{II.4}$$

From the equations (II.1), (II.3) and (II.4) we get,

$$\alpha = 360 \frac{(\pi D - f)^2}{\pi D[2\pi D + f(\frac{\pi}{2} - 2)]} \tag{II.5}$$

thus, all the angles α, β, v are expressed in terms of f and D.

To find the best relation between f and D :

The most desirable aim is that the magnitude of A_f or A_f should be the lowest.

The acceleration A_f in terms of f and D can be expressed as,

$$A_f = 90 \times D \times \frac{\pi D[2\pi D + f(\frac{\pi}{2} - 2)]}{(\pi D - f)^2} \times \frac{1}{360}$$

To find maximum or minimum of A_f,

$$\frac{dA_f}{dD} = 0, \quad \text{which gives}$$

$$\frac{\pi D}{f} \quad 1.5 \pm 0.32$$

evidently positive sign is used.

The length of bed travel with constant velocity should be greater than the length of printing area on cylinder. So,

$$\frac{\text{Circumference of the printing cylinder} = \pi D}{\text{Length of printing area} = f} > 1.5 + 0.32$$

Let $\frac{\pi D}{f} = 3$, $D = 8$ units.

which gives

$$D = 8 \text{ units}$$

$$f = 8.37758 \text{ units}$$

$$S = 3.82949 \text{ units}$$

$$\alpha^\circ = 86.16366^\circ$$

$$\beta^\circ = 153.83638^\circ$$

$$v^\circ = 120.00000^\circ \, .$$

For Example No.2 of Section 4.2 :

In this example design of a disc cam with an oscillating follower is discussed.

Nomenclature used in this example is as follows :

β_1° - period of acceleration during the rise stroke

β_2° - period of the uniform velocity during the rise stroke

β_3° - period of deceleration during the rise stroke

β_4° - period of the return stroke

L_1° - displacement of the follower during β_1° of rotation of cam

L_2° - displacement of the follower during β_2° of rotation of cam

L_3° - displacement of the follower during β_3° of rotation of cam

L_4° - displacement of the follower during β_4° of rotation of cam

and

$$L_4^o = L_1^o + L_2^o + L_3^o$$

Input-output motion curves selected are as follows :

(i) Rise (acceleration) - Half cycloid lift curve (C11)

(ii) Rise (uniform velocity)-Straight line lift curve

(iii) Rise (deceleration) - Half harmonic lift curve (H12)

(iv) Fall - Eighth-order polynomial lift curve (820)

(v) Dwell - -

Calculations :

1. $L_4 = L_1 + L_2 + L_3$ (II.6)

2. The uniform velocity, $v_{cv} = L_2/\beta_2$ (II.7)

3. To match the velocities at the end of Rise (acceleration) and at the beginning of Rise (uniform velocity),

$$\frac{2L_1}{\beta_1} = \frac{L_2}{\beta_2} \qquad \text{(II.8)}$$

4. To match the velocities at the end of Rise (uniform velocity) and at the beginning of Rise (deceleration),

$$\frac{\pi}{2} \cdot \frac{L_3}{\beta_3} = \frac{L_2}{\beta_2} \qquad \text{(II.9)}$$

5. To match the magnitude of deceleration at the end of Rise (deceleration) and the magnitude of acceleration at the beginning of Fall,

$$\frac{\pi^2}{4} \cdot \frac{L_3}{\beta_3^2} = 5.26830\,\frac{L_4}{\beta_4^2} \qquad \text{(II.10)}$$

6. $\beta_1 + \beta_2 + \beta_3\ \beta + \beta$ = total angle specified (II.11)

Let, total angle specified = 300^o,

$L_1 = 5^o$

$L_2 = 20^o$ and

$L_3 = 5^o$.

Putting these values in equations (II.6), (II.7), (II.8), (II.9), (II.10) and (II.11), we get,

$\beta_1 = 45.47847^o$

$\beta_2 = 90.95695^o$

$\beta_3 = 35.71871^o$

$\beta_4 = 127.84586^o$.